# Path Analysis. <br> A Bicycle Rear Suspension Analysis Method. <br> Theory, text, illustrations, and editing by Ken Sasaki. <br> 4-bar path analysis by Peter Ejvinsson. <br> Spanish Version translated by Antonio Osuna. <br> "Linkage" suspension simulation by Gergely Kovacs. 

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About the Authors.

# Path Analysis. <br> <br> Chapter I - Primary Concerns. 

 <br> <br> Chapter I - Primary Concerns.}

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## Objectives.

Path Analysis (PA) is a qualitative method for analyzing the pedaling, braking, and shock absorption characteristics of full suspension frames. The objective is to allow anyone to determine the true merits of any suspension design claims with regard to these characteristics. While the principles here apply in general, we have focused on the non-URT (this usually means bottom bracket on the front triangle and we will use the term to mean so in this work unless otherwise stated), since these designs constitute the lion's share of bikes produced these days.

Most theories on bicycle suspensions one sees attempt to find "the proper pivot point or points" which will make the frame shock non-reactive to pedaling at equilibrium or "sag" (in fact, it is impossible for any frame geometry to do this either exactly or universally, and getting close in any particular case will introduce other problems). A precise quantitative treatment of suspension geometries is a very involved process that requires significant assumptions, even in the most simple of cases. A number of simple theories purport to find correct geometries that eliminate rear shock activation at sag, but to this author's knowledge, none are sound (this excludes the Giant NRS which is meant to be run with no sag). We look at a few of these at the end of the work to demonstrate how PA can be used.

The consumer should be most concerned about getting past industry hype. So rather then spending a lot of time trying to identify proper pivot locations and so forth (beyond an intuitive understanding), we will focus on the issue of what performance characteristics are achievable with viable suspension designs and how one might achieve them in principle. This analysis method is intended as a consumer tool that will allow one to accurately judge marketing claims, as well as the relative merits of suspension designs and theories.

We want this work to be useful to those with absolutely no technical background, so we present the "Main Conclusions" up front in this first chapter. Those with no technical background should also find the "Bogus Marketing" section completely accessible.

In chapters II, III, IV, and $\mathbf{V}$ we have included recommendations for reading and difficulty for each numbered section, stating:

## Read this section if:

You wish to accomplish "this or that" objectives. This section is of "such and such" importance.

## Skip this section if:

You are not interested in "this or that" objectives.
This section is "stated difficulty rating".
Here, "stated difficulty rating" varies among: Not difficult, less difficult, moderately difficult, more difficult, and most difficult. The level of difficulty is referenced to a typical person with about a year of good solid college physics.

We hope that this will help those with a less technical background navigate around the more difficult and less necessary sections. Those with a less technical background should still find the not difficult and less difficult sections, as well as the conclusions from all sections, understandable.

## About the second chapter, "Some Useful Suspension-Related Mechanics":

We have chosen methods of analysis with an eye toward keeping math to a minimum, but some basic physics knowledge is unavoidable.

Much of this chapter is not necessary if one merely wishes to use Path Analysis to evaluate different suspension designs (which is the main intent of this paper).

For this application, we strongly recommend only the "Reference Frames" section.

We have included this chapter mainly as a physics primer for those who wish to rigorously verify Path Analysis and delve more deeply into bicycle suspension physics. If one wishes to independently verify the validity of PA, the "Some Important Concepts" section is most critical to understand. The concluding statements in "An Intuitive Look at Forces and Torques" are also of value. We reemphasize; much of the rest of this chapter is related to Path Analysis only in establishing finer points and is not truly necessary (although, the knowledge will be useful to anyone contemplating bicycle design and some of it will make PA more accessible).

One will also need certain concepts from this chapter if one wishes to completely understand certain flaws in some of the theories detailed in the fifth chapter, "Flawed Theories and Bogus Marketing". Most important among these is the "Center of Mass" (CM) concept, as applied to forces acting through the various wheels and cogs within a bicycle. To this author's knowledge, this concept has previously been unknown in the bicycle industry.

About the third chapter, "Path Analysis":
This is where the main theory is presented. We don't consider any of the sections to be more then moderately difficult. We suggest that all readers read all sections, even if some things are not clear.

About the fourth chapter, "Wheel Path Analyses of Some Existing Models":
This chapter contains the cad drawings by Peter Ejvinsson. These drawings are most informative in conveying information about what is out there at the present time.

Most of the major design types that are more then trivial to evaluate are covered. For the most part, the material in this chapter is extremely easy to understand, the one exception being parts of the "The Virtual Pivot Point (VPP)" section.

In addition, a "Linkage data" link to Gergely Kovacs' "Linkage" suspension simulation program (see below) is provided in each frame's section (for which ltx data files have been pre-made). This program displays the most important characteristics of each frame. Clicking on the link in each frame's section will automatically bring up data on that frame (note that the Linkage program must first be installed, again, see below).

The original motivation for the production of this work was the ubiquity of false theories emanating from bicycle manufacturers and industry magazines, and circulating in bicycle-related web sites. We have thus devoted considerable space to demonstrating the flaws in some well-known and widely accepted theories.

Some of the false theories and marketing are associated with well-known names. This has made the work somewhat controversial. However, we note here, as well as in the chapter, that in all cases involving false theories, vigorous efforts were made to contact and discuss matters with the associated parties, before the release of this work.

One of the oldest and most respected of full suspension frame manufacturers has warned this author that the bike industry is very small and generally not kind to "realists". He also warned that some "retaliation" should be expected and indeed, there has been some.

We are committed to exposing industry hype and nonsense, and to giving the public the best possible chance to make informed decisions, so we will not be deterred by retaliation. While we feel it unfortunate that some of this information has caused a good deal of consternation to some who have already made some very expensive purchases, we will continue with the circulation of this information for the greater public good.

We also note that, generally, the feedback from the industry has been positive; including, we are told, positive comments from one of Renault's senior suspension engineers.

## About the "Glossary":

At this time, the "Glossary" has been done to explain terms in this first "Primary Concerns" chapter that may not be familiar to those new to mountain biking. We have not provided a detailed account of scientific terms in the later sections because of time constraints. We hope that those venturing into these sections will have adequate prior knowledge or know how to obtain such knowledge from more fundamental sources, or that the bold-written essential information will suffice to give a reasonable understanding. In the future, we hope to provide a more detailed account of scientific terminology.

About the "Linkage" suspension simulation program:
Linkage has been created by Gergely Kovacs to produce the most important information about any 4 -bar rear suspension that one might want to concider.

A version of linkage has been included as part of the PA package. The Linkage2 software as well as the source code are also downloadable from the Linkage web site at http://www.angelfire.com/jazz/linkage/. The Linkage web site may contain a more updated version of linkage, since Gergely maintains that site personally.

To use Linkage, one must first download the self extracting zip file and instal the program. This can be done by clicking on any of the "Linkage" links in this page. One may then view suspension designs that are currently on the market or input the dimensions for any other linkage configuration that one desires.

The combination of theory presented in the text here and this program should allow any user to develope a keen intuition for comparing the pedaling and shock absorbtion characteristics of almost all full suspension designs.

The instructions for using Linkage may be found in the program by clicking on the "some help" button.

To install Linkage, click on one of these "Linkage" links. You will be guided through a series of dialogue boxes. The installation defaults will install the Linkage folder, containing the program and ltx data files to C:\Program Files\Linkage2. If you like, you can specify another location. Shortcuts to various Linkage entities, as well as the PA web page will also be installed to the "Start" menu, under "Programs".

After the Linkage program has been installed, Linkage data may be called up in three different ways:

1) 2) As noted above, the Linkage program has been integrated into the text in each frame's section in Chapter IV, for which an "Itx data file" (see the instructions) has been made. By clicking on the "Linkage data" link, the most important characteristics of the frame in question will be shown.
1) 2) One may go into the Linkage 2 directory, which is created on one's computer upon installation, and call the program by clicking on the Linkage.exe icon. One may then open an existing ltx file by clicking the "Open" button or create a new file by clicking the "New" button.
1) 3) One may go into the Linkage2 directory and click directly on an ltx data file icon for a frame of interest.

## About the Authors:

The authors are all avid bikers, who also have technical and/or language skills. We have provided this work freely to the public with the hope that it will benefit consumers and others interested in the workings of bicycles. A short biography and picture may be found on the "About the Authors" page, for those authors who have provided the information.

The authors wish to thank Prof. Curtis Collins, Ola Helenius (Ola H.), and Ray Scruggs (Derby) for their kind suggestions and help in finding errors. Thanks also to Drakon El Elfo for working on the link structure to the Spanish version.

The authors would like to especially thank J.I. Baeza (aka Sikander) and Jose R. Rubio, the editor of http://www.mtbcomprador.com/, for hosting both the English and Spanish versions of this work.

This work will be updated from time to time to reflect current technology. Updated versions will be posted on the above web sites at the links below:

The English version may be found at:

## http://perso.wanadoo.es/jibsna/mtb susp_en

and

## http://www.mtbcomprador.com/pa/english.

The Spanish version may be found at:

## http://www.mtbcomprador.com/pa/spanish

and

## http://perso.wanadoo.es/jibsna/mtb susp_sp.

A link to the above pages, as well as other works by Ken Sasaki, may be found at Http://www.physics.ucsb.edu/~sasaki

We welcome and appreciate all accounts of errors and suggested additions regarding this work that anyone cares to send (bicycle physics@yahoo.com). We apologize if we are unable to answer all correspondence, due to time constraints. Thank you.

## Main Conclusions.

1) 2) All measures of suspension performance depend almost entirely on the paths of the following components relative to any reference frame defined by one of the bicycle frame members: Handlebars, seat, bottom bracket (BB), front and rear wheel axles, shock mounts, and rear brake.

This is the central idea of this work and is explored in the third chapter, "Path Analysis". The following are the main conclusions that may be drawn from this statement.
A) A) General Comments.

The first thing that most people are concerned about with dual suspension bikes is efficiency under pedaling; generally assuming a seated rider position and a bike on smooth ground. But there are compromises that must be made in trying to attain this goal and most other goals associated with dual suspension performance. In reality, the right geometry for any one person will depend on that person's body type (mass distribution), riding style, sensitivity to various phenomena associated with dual suspension motion (such as bump feedback), desired ride quality (comfort, efficiency, etc.), and even the type of terrain in one's backyard.

No geometry is right for everyone and no frame can achieve for any one person every goal generally desired in dual suspension performance.

At suspension equilibrium (natural sag) or any other position in travel, any of the common suspension "types" (mono-pivots, various 4-bar configurations, etc.) can be as non-reactive to pedaling as any other during seated pedaling. However, no geometry can be completely "neutral" throughout a pedaling cycle, without friction. The deviation from neutral can be made small and a good suspension geometry with the right amount of friction can effectively limit oscillations, while remaining supple enough to absorb significant bumps. However, we note that no geometry is perfect in this respect as a warning against all theories purporting to provide a neutral geometry, in principle, without any qualifications.

A word on marketing:
No manufacturer of a bike or frame designed to run at sag (some bikes such as the Giant NRS are meant to run at no sag) is going to market its products with a valid, quantitative theory for constructing rear suspension geometry - telling you why their pivots or whatever are in the right place. The ideas and formulae would simply be too complicated to make a good marketing tool. This author has never seen a valid, quantitative, run-at-sag suspension theory put out by any
company, though quite a number (some of which we examine later in chapter five, "Flawed Theories and Bogus Marketing") market bikes under dubious claims and false theories - some asserting that you can have it all. If any manufacturer or sales person tells you that you can have it all, run away!

Our advice is to ignore all suspension theories and other claims put forth by frame manufacturers and industry magazines, and base your buying decisions exclusively on experimentation. That is, make your decisions by test riding the bikes, even if it is just a parking lot test (you can get a lot from a parking lot test). Ignore all marketing!
B) B) Non-URTs.

Almost all dual suspension bicycles these days are non-URTs. The following comments will apply to these designs, as well as the mono-pivot-equivalent $\underline{i-}$ Drive, made by the GT bicycle company.

The most important thing to look at in assessing any non-URT frame's potential performance is the path that the rear wheel axle travels relative to the main triangle, as the suspension compresses (the main triangle is defined by the seat, handlebars, and bottom bracket). The mechanism that produces the path is not important beyond helping one determine what the path is. In particular, mechanisms that produce similar paths will perform similarly ("suspension rate" aside).

At any moment in time, the rear axle path tangent will primarily determine suspension performance. In particular, within any small segment of any non-URT suspension's travel, that suspension will behave like some monopivot with main pivot that gives the same path tangent.

Suspension rate (spring stiffening) is also significant to suspension performance and is determined for the most part by the paths that the shock mounts travel relative to the main triangle. Shorter travel suspensions tend to have higher rates that increase more drastically as the suspensions move through travel (in part due to the fact that many use air shocks these days). However, most frames mate well with their stock shocks, and all common suspension types can achieve the really useful rates (linear or rising). So rate is only a real issue for those wishing to swap different shocks in and out of a given frame.

All this is not to say that all non-URTs will behave the same. Different geometries will certainly have different characteristics. But this depends almost entirely on the specified component paths. It does not depend on the suspension type.

For example, 4-bar "A" may have a rear axle path curvature substantially different from 4-bar "B", yet A's path may be very close to mono-pivot "C" (circular, with a particular radius and center). Under pedaling and shock absorption, A and C will perform similarly to each other, but differently from B (suspension rate aside).

Almost all non-URTs on the market today have circular rear axle paths out to two or three decimal places, in inches. As a result, the radius and center of curvature primarily determine suspension performance in most non-URTs.

The majority of rear axle paths are of similar radius to conventional monopivots. The 4-bar paths plotted in "Typical Horst Link Designs", which encompass most of the major chain stay pivot design configurations, are all very circular and of conventional radius.

The Giant NRS, The Rocky Mountain ETS-X70, and Cannondale Scalpel have tight radii of curvature, centered inside the rear wheel radius. The Giant NRS and The Rocky Mountain ETS-X70 achieve this through their link configurations. The Scalpel achieves this through the localized bending of its chainstays, about half way between the BB and rear axle. The Scalpel mimics a design we proposed some time ago, called the split-pivot mono. "Soft-tail" designs also have tight radii of curvature, but we consider the length of travel too short for this consideration to be of significance in these designs.

4-bar designs with closely spaced pivots near the frame center can achieve significantly variable path curvature. At the moment, the "The Virtual Pivot Point (VPP)" concept, conceived by Outland and soon to be re-introduced by Santa Cruz and Intense, is unique among viable concepts in its capability to produce significantly variable curvature. However, current examples do not take any real advantage of the possibilities.

Closely spaced 4-bar pivots can also achieve wider curvature then is possible in a mono-pivot. The positive travel sections of the current "The Virtual Pivot Point (VPP)" bikes contain such a curvature and the Schwinn Rocket 88 also claims such.

However, closely spaced pivot locations near the highly stressed bottom bracket area may come with a larger tradeoff between weight and durability, as the links and pivots in this area must be more heavily built.

One unique frame design, which we have yet to evaluate, is the Maverick. At the moment, it is very expensive and still hard to find. We hope to include it in these pages soon.
i) i) Paths and Shock Absorption ("coasting" situations).

We handle only coasting situations here, since shock ramifications for pedaling and braking will be handled specifically in those sections.

A bicycle suspension may be suddenly compressed by the ground either through wheel contact with an obstacle such as a rock or from the impact of a drop-off. In general, we believe that a widely curved rear axle path running slightly up and back is the best solution. Tight curves, either circular or varying are generally inferior for shock absorption. However, this deficiency may be mitigated to some degree by having the path tangent tilting backward through all or most of the travel (for example, having a high main pivot, either real or virtual), as is the case in the The Rocky Mountain ETS-X70 and, substantially, the Giant NRS. One might also find that short travel designs such as the Cannondale Scalpel do not have enough travel for this deficiency to be significant.

In the case of a drop-off, the situation is obvious; a linear path will offer the smoothest, most consistent compliance.

In the case of an obstacle, the bump force will be up and back relative to the frame, so the initial tangent should be up and back. The direction of the bump force will turn more vertical as the bike clears objects of "ride-able" size, so a widely curving path turning slightly upward at the top should be ideal.

Experimentation will determine the most desirable path incline and radius of curvature.

Rising suspension rates benefit short travel designs, since this will allow better initial compliance, while reducing the probability of hard bottom-outs.

## ii) ii) Paths and Pedaling.

The rear axle path tangent will determine how the suspension will react to pedaling at any given moment. This means that, neglecting friction in the mechanism, each particular geometry will have its maximum effectiveness only in certain "ideal" gears (from a practical standpoint, this could mean one gear or several). The further the gearing from ideal, the more reactive any suspension geometry will be.

For a given deviation away from ideal gearing, "suspension rate" (spring stiffening) will determine the amount of reaction from a pedal stroke. Shorter travel suspensions tend to be less reactive to pedaling then longer travel versions, since short travel designs should have higher, more rising rates. However, the difference between linear and rising rates will be small in the shallow regions of travel where pedaling is affected. In practice, the actual
rates in these shallow regions will largely be a function of the total travel, or rear axle path length.

It is common these days for designers to take into account the slight tendency of a bike to fold or "squat" under acceleration. To do this, one adjusts the rear axle path so as to increase, by some significant amount, the distance of the rear axle from the bottom bracket (BB) as the suspension compresses. This allows chain tension (mainly) to counter the squat. But this also creates significant bump feedback. We want to be clear on one thing: There is no free lunch here. Have an increasing effective chain length between the cogs - get some degree of bump feedback.

The one area where some multi-links (this usually means 4-bars) may have a slight benefit over conventional mono-pivots is in bump feedback to the pedals.

4-bars offer the possibility of both variable curvature and tight, circular curvature as the rear axle moves relative to the main triangle. Both of these possibilities allow for a center of curvature inside the rear wheel radius. Tight curvature above equilibrium allows the suspension to counter squat at equilibrium, while more effectively limiting feedback. Bikes with tight circular curvature should be run with little or no sag to prevent problems from feedback under suspension extension.

However, as we have noted, 4-bars on the market today do not provide significantly varying curvature and only the Giant NRS, The Rocky Mountain ETS-X70, and Cannondale Scalpel, have significantly tighter curvature (though one might find that the ETS-X70 does not have a small enough radius, nor the Scalpel enough travel for this to be significant for him or her, with regard to pedaling).

This means that almost all non-URT designs on the market today (with the three exceptions) must make essentially the same compromises between feedback and anti-squat. Some prefer the rearward tilting axle path and the generally increased efficiency provided by the anti-squat. Others prefer the smoother pedaling over bumps provided by a more vertical path tangent at sag.

## iii) iii) Compromises.

We have seen that rearward axle path tangents at equilibrium should offer some advantage while pedaling over smooth terrain and during shock absorption while coasting. However, this will also produce bump feedback while pedaling over bumps. Many riders say that they are very sensitive to this trade off, even to the point where differences of less then an inch in main pivot locations are noticeable. Some prefer the generally efficient rearward tangents, while others
want the smoother pedaling vertical tangents. So we have a compromise with which to deal.

We have also noted that tight curves above equilibrium, whether circular or varying, may help reduce the bump feedback of a rearward tangent. However, curves tight enough to make a significant difference in the shallow regions of travel where riders are likely to be pedaling may produce inferior bump performance deeper into the travel, since wide curvature should be best for shock absorption. Though again, designs with rearward paths through travel, such as the The Rocky Mountain ETS-X70 and, for the most part, the Giant NRS, may mitigate this compromise to one degree or another.

Furthermore, while variable curvature has its allure, the potential for a bigger tradeoff between weight and durability in comparison to conventional 4-bars, due to closely spaced pivots near the bottom bracket, shows that variable curvature designs may not be without their compromises (this in addition to the compromises noted above involving tight curvature).

This furthers a theme that we revisit throughout this work - there are no "right paths" or "right pivot points". We have seen this in the mass distribution considerations of having riders with different body types. We have seen this in the fact that no geometry can be completely non-reactive through a pedal stroke, without the help of friction. And now we see it again in the fact that there are choices that must be made, depending on what type of suspension performance one wants.

Human beings can be surprisingly sensitive to physical situations. The author finds that a difference of just two millimeters in the height of a road bike seat gives the feel of a completely different bike. So we are not surprised to find that some people hold small geometric differences as important and we must assume these positions to be legitimate.

However, we must note that some people experience "have-it-all" performance in some designs from manufacturers that claim such performance (though this is certainly not the case with most experienced riders that this author has encountered). Since we have seen that have-it-all performance is impossible, we must conclude that either the powerful psychosomatic phenomenon is at work or that some of the considerations that we have been exploring are not all that discernable to some people, or perhaps it is a little of both.

All of this makes the question of suspension performance largely philosophical. So to continue another theme, we again suggest that test riding be done to determine what performance characteristics are right or even discernable for each person, even if it is just a parking lot test (you can get a lot from a parking lot test).

In the final analysis, none of the major suspension types has a clear advantage over the others. There are lots of happy mono-pivot owners out there (including those with mono-pivot-like Ventanas and Rocky Elements) and there are lots of happy 4-bar owners out there. This pretty much says it all.

## iv) iv) Paths and Braking.

The biggest question regarding braking in dual suspension bikes is whether or not 4 -bars rear-brake better then mono-pivots.

There is a very well established myth (well-propagated by the magazines) that mono-pivot shocks will lock under rear braking. This is known as "Brake Induced Shock Lockout" or BISL. We have demonstrated this to be false.

We have also demonstrated through experiment that mono-pivots do not significantly extend or compress under braking.

We have even demonstrated that certain 4-bar designs, such as the Jamis Dakars and the Psycle Werks Wild Hare, should brake almost exactly the same as a mono-pivot with identical main pivot location. We have seen many Dakar and Wild Hare reviews from Mountain Bike Action, Bicycling, and several other industry magazines. That none of the reviews mentions BISL in these 4-bar designs is a good indication that it does not actually exist.

Most 4-bars extend from natural sag under smooth-surface braking a bit more then equivalently main pivoted mono-pivots, establishing a new equilibrium position and rate. Some, such as the Yeti AS-R, compress under smoothsurface braking relative to equivalently main pivoted mono-pivots. In addition, changing frame geometry through travel, due for example to bump compression, may cause the braking effect to change, further altering the effective suspension rate of a 4-bar.

But again, none of this leads to the conclusion that 4-bars brake better then mono-pivots in general, since a mono-pivot could well have (and some probably do) the same rate under both braking and pedaling as most 4 -bars under braking.

The biggest consideration is the relation of the rider's body mass to the wheels and what it will do under braking. This author believes that between most of the designs, the differences are just not enough to merit a general statement.

Some people find 4-bars to brake better, but others do not, though we have seen no double-blind tests. In the end, the small differences between some designs
may be significant enough for some people to feel a difference. But in general, we suspect that this is again just a case of the very well established psychosomatic phenomenon. This would not be the first time that people have been told that something is so and many have experienced what they expect (this is why placebos cure illness). Or perhaps it is again a little of both.

We also have no doubt that the BISL myth has been propagated by some in the interest of selling more expensive 4 -bar designs. We see no $\$ 2,000$ monopivots.

In the near future, we hope to do a double-blind experiment to see once and for all if there is a difference between 4-bars and mono-pivots, under braking. We will publish any results in subsequent editions of this work.

In any case, our advice here, as always, is to make your decisions through testing the bikes, if possible.

What we have stated above regarding 4-bar linkages also applies to floating rear brake systems, since a floating brake will give a bike the same rear braking character as a 4-bar with the linkage geometry of the floating brake. Imparting the character of its linkage geometry is the only thing a floating rear brake does, for good or ill. This will, for example, give a typical mono-pivot suspension the tendency to extend under braking, rather then its inherently neutral character.
C) C) An Open Letter.

I would like to close this section with a segment from an open letter I published some time ago:

You will find people who worship most major designs out there and others who despise these same. There is a good reason for this. Most of it only exists in people's minds. Many people hate mono-pivots, but revere Ventanas, not knowing that a Ventana is essentially a mono-pivot with linkages that act as shock tuning (with respect to pedaling at least). There are some differences in the major designs, and some small advantages here and there, but in the end it is mostly academic. Stick with the major concepts and one will not change your life over the others. Of course I am speaking of comparing bikes within particular categories, not comparing free ride to XC or downhill.

This is the conclusion I have drawn from my model. Most people build a model and use it to sell one idea or another. My contention is that the four or five major, basic forms CAN all work about as well as the others.

In the end, execution is far more important. A quality company is far more important then a slight difference in the position of a pivot. And make sure the
bike fits right. This applies to intended use (be realistic) as well as body weight and body dimensions.

Most of us with propellers on our heads just like to talk about this stuff because we enjoy applying the skills we have acquired to our hobby (though some obviously have religion). Let me close with a piece of a conversation I had with a professor of mechanical engineering, with whom I have discussed my theory several times:

I said that in the end, the simplest designs are the best. He responded, that this is almost always the case.

For most people this means a basic mono-pivot, or a basic 4-bar with the pivot on the chain stay or the seat stay. Some are obviously swearing that you need a link on the chain stay, but don't tell that to people who own Ventanas or Rockies.

Keep it simple and go with your gut feeling; you have to like the bike when you look at it (with whatever standards you really find important).

Good luck,
Ken Sasaki.


# Path Analysis. <br> Chapter II - Some Useful Suspension-related Mechanics. <br> Theory, text, illustrations, and editing by Ken Sasaki. <br> 4-bar path analysis by Peter Ejvinsson. <br> Spanish Version translated by Antonio Osuna. <br> "Linkage" suspension simulation by Gergely Kovacs. 

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## Some Important Concepts.

## Read this section if:

You want to verify for yourself the validity of the "Path Analysis Main Assertions." and understand some of the related analysis in the fifth chapter, "Flawed Theories and Bogus Marketing.".

We strongly recommend at least reading the "Reference Frames" subsection. It will be very useful to understand this basic physical concept in later sections.

Skip this section (except the "Reference Frames" subsection) if:
You will accept the Path Analysis main assertions and are just interested in using PA to make conclusions about what suspensions can do and comparisons between various bikes.

This section is moderately difficult.

Fully understanding PA for bicycles requires some important concepts. We strongly suggest that those wishing to fully understand PA spend some time becoming familiar with these concepts as most erroneous suspension theories involve the neglect or misunderstanding of one or more of these, "Center of Mass" in particular.
A) A) "Reference Frames".

In order to analyze any physical situation, we must create a reference frame. This is usually represented by of a set of coordinates in space, consisting of a mutually perpendicular set of lines, or "axes", with common intersection. The place where the axes cross defines the origin, or zero point. We usually give names to each axis, such as "x-axis" or " $y$-axis". Depending on the sort of information in which we are interested, coordinates could consist of one, two, three, or even more axes (though more then three axes depict more then the normal spatial dimensions and obviously cannot be pictured).

Often, we assign units of length along each axis. Each point in space lies along a line perpendicular to any given axis. Points may thus be defined by the set of numbers corresponding to the points along the axes through which these perpendicular lines pass. This system is called a rectangular coordinate system.

Figure 2.1) shows a 2-dimensional rectangular coordinate system. The axes are colored gold, with black unit markers indicating length (exactly what the length scale is in this case is not important, but usually it will be specified). A particular point $(3,-2)$ is noted in the lower right quadrant. The x-coordinate is usually specified first, as it is here. A bicycle frame (and some other things that we don't need to worry about at this point) is pictured in the coordinate system, with the main pivot located at the origin.

Figure 2.1)


Reference frames may be defined by objects such as the earth or pieces of a bicycle frame. That is, we treat our coordinate system as if it were attached to the defining object. If the defining object is undergoing a linear acceleration, or has angular velocity (meaning it is rotating), then so will the reference frame. In this case, we call the reference frame "non-inertial" or "accelerated".

If the coordinate system in Figure 2.1) were attached to the earth, then over time, the pictured frame (as part of a bicycle) would move with respect to the coordinate system. Since the earth is only undergoing very small accelerations we consider it (essentially) an inertial reference frame, for most practical purposes. If the coordinate system were attached to the main triangle, then the positions of other objects, such as the rear suspension members, would be defined by how they move about the main triangle. Since the main triangle is often undergoing significant accelerations, we consider it a non-inertial reference.

One thing to note is that in non-inertial reference frames, fictitious forces and torques can appear due to frame acceleration, the most well known of which is the "centrifugal" force of a rotating frame. If one is riding on one of those carnival rides that spin round and round, one feels as if there is a force (like gravity) pulling one out from the center of rotation and pinning one up against the constraining wall of the ride. This is the centrifugal force, which is only apparent.

The centrifugal force should not be confused with the "centripetal" force, which is the force of the wall causing one to deviate from a linear path and thus to rotate in a circle. The centripetal force is a real force. The centripetal force acts on the rider and points in, towards the center of rotation. The centrifugal force
seems to act on the rider in a direction pointing out, away from the center of rotation.

Sometimes it is only important to define a reference by some object, but not important to define where the origin is located or any length scale. In this case, we may define the reference frame by naming some object, without specify anything else. For example, we may specify the reference frame of the bicycle main triangle. We do this when we want to consider how other objects move with respect to the object defining the reference, but don't care about particular distance scales and so forth.
B) B) "Degrees of Freedom".

Each degree of freedom denotes an independent way in which a body can move. A completely free body has six degrees of freedom. Given standard rectangular coordinates, a free body can translate in any of the three coordinate directions and it can rotate around the three coordinate directions.

PA makes use of the degree of freedom limitations on bicycle components. For example, in the reference frame of the ground, a dual suspension bicycle main triangle has three relevant degrees of freedom while the bike is traveling in a straight line. It can translate horizontally and vertically, and it can rotate, all in the plane defined by the rear wheel. The balance of the rider limits the other degrees of freedom. If we fix the main triangle in space, relevant bicycle components only have at most one degree of freedom.
C) C) "Nature Varies Smoothly" (NVS).

The equations describing the laws of nature are continuous relations (usually stated as functions). The value on one side will not jump discontinuously as the parameters on the other side vary continuously.
(This excluding the quantum realm - very small, very big, very cold, etc.)
As a result, if we imagine a pivot position varying smoothly along some arm in a mechanism, the equations of motion will vary smoothly also. That is, the physical situation (laws) will not jump at some point. The way the mechanism behaves will change continuously.
D) D) "Approximation".

One of the most difficult things physics students have to grasp is when and how to make approximations. The simplest form of approximation is that involving quantities much larger or smaller then other relevant quantities in a given
physical situation. We will give two examples of proper and improper approximations by this method, using mass.

Consider the mass "M1" which is much, much bigger then the non-zero masses "M2" and "M3" in Figure 2.2 A). The masses move without friction in one dimension. In considering the motions of all bodies, approximating M2 and M3 as having zero mass would not be useful, since information about the interactions of the small bodies would be eliminated. On the other hand, if we approximate M1 as infinite, useful calculations may still be done (this is often done when considering human-sized objects interacting on the earth).

In Figure 2.2 B ), we have the opposite situation from Figure 2.2 A ). Here, we might very well neglect the mass of M2 in quantifying the results of M1 colliding with both M2 and M3.

These two examples show that, in certain situations, the odd entity may be approximated.

Figure 2.2)


Next, consider Figure 2.3 A), which depicts a block of mass M connected by a rope to a suspended wheel that is of very small mass relative to the block and spins with negligible friction. If we want to quantify what happens when the block is suddenly dropped, we could not ignore the mass of the wheel, since our approximations would then describe a non-physical situation - namely the unconstrained angular acceleration of a zero-mass body. On the other hand, if we were to ask the angular acceleration of the wheel in the "Atwood" device in Figure 2.3 B ), we could get a pretty good answer while ignoring the mass of the wheel, if both M and m are large compared to the wheel.

For a further discussion of this topic, see the "Mass Approximation." section in "P.A. Basics."

Figure 2.3)

A

E) E) "Center of Mass" (CM).

The CM of a solid body, or system of bodies, is the weighted average, spatial distribution of all mass in the system.

For example, the CM of a symmetric object, such as a wheel, is at the center or axle.

For us, the most important fact regarding the center of mass is that a force applied to any part of the body will cause a parallel acceleration at the center of mass. For example, a force applied to a wheel somewhere along its radius, in the plane of the wheel, will cause acceleration at the axle parallel to the force. For a wheel in free space, this means that the wheel will start translating in the direction of the external force, as well as rotating. Figure 2.4) shows this situation.

Figure 2.4)

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To get an idea of how this might be applied, consider the following question, which I call "The Pole and Wheel":

A pole is attached to the ground via a hinge with negligible friction. A wheel of mass $m$ is attached to the top of the pole via an axle that also has no significant friction. A rope of negligible mass is wound around the wheel's circumference with the end hanging toward the ground on the right hand side of the wheel. All of this is balanced at equilibrium, with the pole pointing vertically from the ground (what we have here is really just the rear of a mono-pivot attached to the ground).

How should you pull on the rope, in order that the pole will not fall? Should you pull vertically; left or right of vertical? Should you pull through the pivot?

[^0]Figure 2.5)


The key is to realize that the tension in the rope induces a force at the edge of the wheel, which in turn will induce a parallel force at the axle, just as theory predicts. The result follows.

One may look at this system as a mono-pivot bicycle with the earth as a giant front triangle.
Note: The Pole and Wheel question has been extremely difficult for most people. Even most physics professors do not get it right the first time, and none of the well-known bicycle suspension designers has realized this in the past. However, if one wishes to understand the forces present within a pedaled bicycle, this concept is essential.

If the reader is having difficulty with this issue, our suggestion is to conduct the experiment. A good way to do this is to take the front wheel holder from a Yakima (or other) car-mounted bike rack and remove all of the hardware. This is your pole. Attach a bike wheel and conduct the experiment. You will see that if the line of force at the wheel's edge goes though the bottom of the pole, then the pole and wheel will topple over. If the line of force at the wheel's edge is essentially vertical, then the pole and wheel will not topple over.

Those wishing a full presentation of the methods used here can find one in "Classical Dynamics of Particles and Systems", by Marion, 1970. The center of mass equation of motion due to an external force is on page 68. The derivation starts on page 67.

In the previous example, we have considered a single force on the wheel, with the rotational inertia of the wheel opposing the external force. However, we may also consider multiple forces acting on the wheel. If there are at least two forces creating opposing torques on the wheel, about the axle, and the mass of the wheel is small compared to other quantities, then we may ignore the wheel mass. The simplest example is that of the Atwood machine in Figure 2.3 B). While the machine is in motion, we may take the tension T in the rope on each side to be equal, if we neglect the mass of the wheel. The force at the wheel axle is 2 T ; both force vectors external to the wheel pointing in the same direction. If the external forces on the wheel are not pointing in the same direction, the total force at the axle will be the vector sum.

Figure 2.6) shows the forces acting at the axle of a negligible mass wheel, which is experiencing multiple external forces at two different radii.

Figure 2.6)


Later, in Figure 2.12) of "An Intuitive Look at Forces and Torques.", we will consider an example of this, with the crank being an example of the wheel, and the pedal stroke from the rider and chain tension being the external forces. Those wishing to understand the calculations associated with Figure 2.12) should keep Figure 2.6 in mind.

1) F) "Coaxial Condition"

If a wheel or a crank is mounted coaxially to a pivot in some mechanism, it does not matter how the object is mounted physically. In a bicycle, the rear wheel could be physically mounted to the seat stay or chain stay, and the crank could be mounted to the main triangle or the seat stay - none of this matters. The physical situation will be the same in all cases as long as the specified objects and pivots are coaxial.
F) G) "Instant Center" (IC).

Imagine a mechanism that has two rigid components (possibly among other things). Two rigid arms, attached to the components by pivots, connect these two component sides. An example would be a 4-bar suspension linkage. In this case, one component could be the main triangle and the other the rear link.

Next, fix a reference frame to the first component, which in our example is the main triangle. At any given time when the arms and the other component (rear link) move about the main triangle, we can calculate the path tangents for all points in motion on these objects by constructing the IC. We do this by drawing lines through the two pivots on either side of each arm. If the arms are linear structures, then the axes will determine our lines. The point where the
two lines cross is the IC. The path tangent of any point in motion is perpendicular to the line between the IC and the point (obviously all of this is in a single plane).

Figure 2.7) shows four bars, with the red dots representing pivots. The light blue lines are the line segments defined by the upper and lower pairs of pivots. The black dot at the intersection of the blue lines is the IC of side $B$ and the adjacent arms moving in the reference frame of side $A$. The green mark represents an object (such as a wheel axle) on side B, with the dark blue line representing the object's path tangent as it moves about side A . The dark blue tangent is perpendicular to the line through the IC and green object.

Figure 2.7)


The idea is that for a small angle $\mathrm{d} \theta$, the movements of our two arms produce the same paths as if component B and the arms were rotating about the IC.

WARNING! The IC is not a "virtual pivot". In general, it is constantly in motion, unlike a pivot. In Figure 2.7), as side B moves about side A, the IC will constantly change, as will the dark blue line representing the green object's path tangent. Many errors in suspension theories result from ascribing to an IC the attributes of a pivot. The IC gives useful information, but only for an instant of time, thus the name, instant center. By contrast, a pivot might be referred to as a constant center.

Figure 2.8) depicts a 4-bar suspension frame. The blue lines reveal various instant center positions for the rear suspension in the reference frame of the main triangle. The red curve plots the IC movement as the suspension moves through its travel. If the distance from the rear wheel axle to a fixed point remains almost constant through suspension travel, that is if the rear axle path is almost circular, then we can consider the fixed point a "virtual pivot" for the rear axle. The light green lines in Figure 2.8) reveal that a "virtual pivot" exists for the rear wheel axle, in the reference frame of the front triangle. Note that any virtual pivots will be unique to each point on the rear link. That is, the virtual pivot for the rear axle in this case will not be one for any other points on the rear link of significant distance from the rear axle. Other positions on the
suspension may have virtual pivots, or they may not, since the distance deviation from any fixed points may be too large for useful approximations.

Figure 2.8)


An Intuitive Look at Forces and Torques.

## Read this section if:

You want a semi-qualitative analysis of forces and torques going on within a suspension bicycle. Understanding everything in this section is not important to understanding Path Analysis. This is just for people who want to go a little deeper.

Read just the conclusions in this section (written bold) if:
You want just the conclusions of the analysis for application to other sections. The conclusions should not be too difficult to understand, so we suggest that one at least give them a quick read. Whatever one does not understand probably will not matter too much, but one might pick up some useful information for the trouble.

## Skip this section if:

You are just interested in using Path Analysis to make conclusions and comparisons regarding various bikes.

This section is among the two most difficult in the work.

We here do an intuitive study of forces and torques in a mono-pivot non-URT to understand the things of which a suspension theory must account. This will help us further understand what goes on in a suspension and the limitations on what any viable design can really accomplish. We try to keep the math to a minimum, however we will be making some minimal calculations to demonstrate certain solutions in principle. Those with a less technical background can ignore the calculations and look directly at the conclusions, which are written bold.

The most important lesson of this section is that mass distribution is an important consideration in the physics of full suspension bicycles. No quantitative theory can be correct without this consideration.

It is common practice to take no reaction of the rear shock to pedaling as the goal, so we will follow.

Figure 2.9) shows the front and rear triangles of a "coasting" mono-pivot, with the various forces acting on them, minus friction in the hubs and air, which we neglect (the forces are not drawn to scale). "CM" indicates the rough position for the rider/main triangle center of mass.

Figure 2.9)


All forces sum to zero when there is no pedaling. For this reason, we need examine only those forces and torques that appear as the result of a pedal stroke. Figure 2.10 ) shows the picture without the coasting forces drawn.

Figure 2.10)


There are a number of ways one might go about analyzing this situation. We will use the torque equation:

1) 2) $\mathbf{I} * \alpha=\sum \tau$.

Here, $\mathbf{I}$ is the moment of inertia of the body in question, $\alpha$ is the angular acceleration, and $\sum \tau$ is the sum of the torques on the body. This is the angular analogue to ma $=\sum \mathbf{F}$. Using equation 1), we will examine what issues are involved in keeping the torque balance between the main triangle and swingarm, about the main pivot, as close to zero as possible.

For precise calculation, this method is not very useful, since some of the torques are not easy to state explicitly and all of the torques are time-dependant (all except chain torque depending on the positions and movements of the two frame members, which will change with time through the pedal stroke). We can thus glimpse the complexity of any completely rigorous analysis. But for us and our mainly intuitive study, this method will be very useful, since we can use it to explore a number of interesting points with minimal math.

We start with a comment on chain force.
One must be very careful when thinking about lines of force in that magnitude, direction, and location are all important. Even equivalent gear ratios generally produce lines of force that differ in magnitude and direction as well as location.

Figure 2.11) shows a drive train with two possibilities for a 1-1 gearing. L is the crank lever, $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ are the respective $\operatorname{cog}$ radii, and T 1 and T 2 are the chain tensions for each case.

Figure 2.11)


If a force F is induced at L with resulting tension in the chain (examining one case at a time), the resulting torque equation for the crank is (assuming a nonURT just for ease of calculation):
2) 2) $\mathrm{I} * \alpha=\mathrm{F} * \mathrm{~L}-\mathrm{T} 1 * \mathrm{r}=\mathrm{F} * \mathrm{~L}-\mathrm{T} 2 * \mathrm{R}$

Thus, $\mathbf{T} 1 / \mathbf{R}=\mathbf{T} 2 / \mathbf{r}$. That is, the chain tension decreases as the front $\operatorname{cog}$ radius increases - a rather interesting result. So, even the two $1-1$ situations will generally not produce equivalent results for suspension activation under pedaling. This actually should not surprise us, since the energy transmitted through the system should be the same in both cases. Energy can be expressed as $\mathrm{T} * \mathrm{~d}$, where T is the chain tension and d is the length of the chain that passes by some fixed point like the seat tube. Since a greater chain length is pulled in a bigger chain ring for a given rotation of the crank, we need a lesser force to keep the energy constant.

Continuing: Figure 2.12) shows the diagram for the calculations to follow. The partially pictured triangle represents the main triangle, to which the crank is attached. The lower horizontal line represents the swing arm of length $\mathrm{S}_{\mathrm{L}} . \mathrm{R}$ is the front cog radius and $r$ is the rear cog radius. $L$ is the length of the crank arm. F is the force of a pedal stroke. T is the resulting chain tension. h is the perpendicular distance from the chain to the pivot, D is the perpendicular distance from the pivot to the force line at the BB induced by the chain tension, and $d$ is the perpendicular distance from the pivot to the line through the rear axle that is parallel to the chain tension. $\theta$ is the angle of the chain from the swingarm axis.

Recall the Center of Mass/force phenomenon described in Figure 2.6) of the "Center of Mass" section; it applies both to the proactive force $F$ acting on the crank and to the reactive force $\mathbf{T}$ from the chain, both with results at the center.

We have not pictured all of the forces present on the suspension members, but only those induced as a result of a pedal stroke that are relevant for our calculations.

We assume that the crank axle and main pivot are close together relative to frame size. (A few frames such as the The Rocky Mountain ETS-X70 and the $\underline{i}-$ Drive differ from this significantly, but this will not impact on the relevant points, and all conclusions will be valid for all suspension frames). This allows us to approximate the force at the pivot from the crank axle as if the two were coaxial. We also assume that the crank mass is negligible. This will allow us to equate forces on the chain ring and crank arm to resulting forces at the crank axle.

Figure 2.12)

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In the following calculations, the reference frame for the main triangle torque equation is centered at the suspension main pivot and does not rotate (with respect to the earth). The reference frame for the swing arm torque equation is centered at the rear axle and also does not rotate. Since both reference frames do not rotate, the bodies will stay at a static angle to each other if their angular accelerations are the same (the initial angular velocities are both zero). $\mathrm{I}_{\mathrm{F}}$ and $\mathrm{I}_{\mathrm{S}}$ are, respectively, the main triangle moment of inertia about the main pivot and the swing arm moment of inertia about the rear axle. $\tau_{\mathrm{Fi}}$ and $\tau_{\mathrm{si}}$ denote the various torques on the respective bodies about their coordinate origins, which
include the torques due to the chain, fork/front wheel (friction and inertia), bicycle acceleration (the most commonly recognized of which is squat), the crank rotation and lower rotating parts of the rider's body, and also the torques due to the interactions of the two frame members [we do some further work with these interaction torques in Appendix A) "PCL Problems; Some Further Calculations.", should anyone have questions as to exactly what these are].
\{An aside: One should not be too concerned about the following detail, but the astute reader will note that we are using two different non-inertial reference frames for each bicycle frame member. The bicycle acceleration and interaction torques are the fictitious torques in these reference frames.\}

The torque equations for the rider/main triangle and swing arm are then, respectively:
3) $\mathrm{I}_{\mathrm{F}} * \alpha_{\mathrm{F}}=\sum_{\mathrm{i}} \tau_{\mathrm{Fi}}$
and
4) $I_{S} * \alpha_{S}=\sum_{i} \tau_{S i}$

Achieving the stated goal of minimizing suspension reaction to pedaling generally involves finding the best place for the main pivot relative to the chain force line for an assumed condition (mass distribution, etc.). To do this, one must express the chain torque in terms of $h$ and solve for this quantity in the proper equation. Fortunately the chain and pedal torques are easy to state in equation and will allow us to get a formula in principle for the desired relation of pivot and chain.

Let $\tau_{\mathrm{FC}}$ denote the torque on the main triangle due to pedaling and the resulting chain tension.

Noting that with a negligible crank mass, $\mathrm{F}=\mathrm{T} * \mathrm{R} / \mathrm{L}$, the torque on the main triangle due to the pedal stroke and resulting chain tension is:
5) 5) $\tau_{\mathrm{FC}}=\mathrm{F} * \mathrm{~L}-\mathrm{T} * \mathrm{D}=\mathrm{T} * \mathrm{R}-\mathrm{T} * \mathrm{D}=\mathrm{T} *(\mathrm{R}-\mathrm{D})=\mathrm{T} * \mathrm{~h}$.

So we see that, neglecting the mass of the crank, the torque on the main triangle from pedaling is just as if we had been pulling on the chain from a point on the main triangle that is a perpendicular distance $h$ above the pivot - a very interesting result (see Ola Helenius for an interesting intuitive argument for this result, though we are not exactly sure where it is on his site). However, one must be very careful not to take this result too

## far; as we have seen, for a given pedal force, the larger the radius of the front cog, the lower will be the chain tension.

Let $\tau_{\text {SC }}$ denote the torque on the swingarm from the chain (again, ultimately from pedal force). Again, since in practice the pivot is relatively close to the BB compared to the frame size, we approximate the force on the swing arm at the pivot as that of the chain force induced at the BB (these values will be very close for typical frames). With this approximation, we have:
6) $\tau_{\mathrm{SC}}=\mathrm{S}_{\mathrm{L}} * \mathrm{~T} * \sin (\theta)=\mathrm{S}_{\mathrm{L}} * \mathrm{~T} * \frac{\mathrm{~d}}{\mathrm{~S}_{\mathrm{L}}}=\mathrm{T} * \mathrm{~d}$

To have the torque balance between the main triangle and swingarm about the main pivot equal to zero (to get no reaction of the rear shock), we want the front and rear triangles to rotate en unison - that is, we want the $\alpha_{F}=\alpha_{S}$. Solving for the $\alpha$ 's in equations 3) and 4) and setting the two expressions equal to each other we get:
7) $\frac{\mathrm{T} * \mathrm{~h}+\sum_{\mathrm{i} \neq \mathrm{C}} \tau_{\mathrm{Fi}}}{\mathrm{I}_{\mathrm{F}}}=\frac{\mathrm{T} * \mathrm{~d}+\sum_{\mathrm{i} \neq \mathrm{C}} \tau_{\mathrm{Si}}}{\mathrm{I}_{\mathrm{S}}}=\frac{\mathrm{T} *(\mathrm{r}-\mathrm{h})+\sum_{\mathrm{i} \neq \mathrm{C}} \tau_{\mathrm{Si}}}{\mathrm{I}_{\mathrm{S}}}$.

Now solving for h , we get:
8) $\mathrm{h}=\frac{\mathrm{r}}{\left(1+\frac{\mathrm{I}_{\mathrm{S}}}{\mathrm{I}_{\mathrm{F}}}\right)}+\frac{\sum_{\mathrm{i} \neq \mathrm{C}} \tau_{\mathrm{Si}}}{\mathrm{T} *\left(1+\frac{\mathrm{I}_{\mathrm{S}}}{\mathrm{I}_{\mathrm{F}}}\right)}+\frac{\sum_{\mathrm{i} \neq \mathrm{C}} \tau_{\mathrm{Fi}}}{\mathrm{T} *\left(1+\frac{\mathrm{I}_{\mathrm{F}}}{\mathrm{I}_{\mathrm{S}}}\right)}$.

This is the zero torque balance formula for the main pivot position, relative to the chain line for a non-URT mono-pivot (with pivot not too far from the BB compared to the size of the frame - again, almost always the case).

One might conclude that h depends on T , as T appears in the denominator of the last two terms. We state without proof that T will appear as a factor in all of the torques, just as it did for the chain torque, with the exception of that resulting from the fork. So with the noted fork exception, h does not depend on T. [In "PCL Problems; Some Further Calculations.", we give an example of how the torques, for the most part, eliminate T from equation 8).]

We may draw the following conclusions from equation 8):
First, notice that the moments of inertia for both bodies are in all terms. This tells us that it will be impossible to construct any sort of a quantitative
suspension theory without taking into account mass and its distribution. Mass distribution will be of equal consideration for all other suspension types. This rules out certain "Special Point" Theories, such as the most naive "Pivot at the Chain Line" (PCL) theories.

The second thing we notice is that since the torque values are time-dependant, h will also be time-dependant through the pedal stroke. We thus see that there is no single "proper pivot point" (or points), exactly, through an entire pedal stroke. In addition, we note as a matter of intuition, that as the rider makes a pedal stroke, the system of frame members, on average, will rotate back relative to the rear axle $(\alpha \neq 0)$. Between pedal strokes, the frame members will fall back down, and not in such a way as to keep the rear shock inactive without help from friction in the pivots. This further tells us that it is impossible for any rear suspension geometry to be completely non-reactive to pedaling, without static friction.

The time-dependant nature of our mono-pivot situation is also common to all other suspension types, since frame member orientation changes through the pedal stroke in all of these bikes as well. In particular, monopivots can approximate a zero torque balance about the main pivot as well as any 4-bar, through the pedal stroke.

The effects of changing frame member orientation are relatively small, but we note them as a warning against all theories that purport to completely eliminate shock activation to pedaling, in principle, based on geometry (even if there is an assumption for mass distribution), such as "Special Point" Theories.

Since the frame orientation effects are relatively small, a single geometry can behave relatively uniformly through the pedal stroke. Suspension geometry can thus keep pedal effects on the shock to a minimum, on average, and let friction do the rest. Pedaling effects on the rear shock can be made small compared to any significant bump, so a good suspension with the right amount of friction can effectively control oscillations, while remaining supple enough to absorb any significant bump.

Lastly, as a matter of intuition, we note that in any suspension, the less the rear shock extends during a pedal stroke, the more the front shock extends. There will be loss to friction either way. The ideal proportion of front and rear shock activation will be that which minimizes sympathetic oscillation.

## Read this section if:

You want a Path Analysis perspective on suspension tuning.
Skip this section if:
You are not interested.
This section is more difficult.

We don't want to spend too much time here, since this is probably the last issue about which a consumer should worry. All non-URT types can achieve all of the really useful suspension rates and most frames out there pair up fairly well with the stock shocks. Rate is only really a consideration for those who are real suspension wonks (yes, the author is a suspension wonk and if you are reading this, you are in danger of wonk-hood also) wanting to swap between coil and air shocks, which generally have different internal rates. Pairing a falling rate frame with a linear coil shock or an extremely rising rate frame with an air shock might not have acceptable results.

However, we do refer to suspension rate in other parts of this work, so we will look at the most important considerations.

All springs have "rates" and a suspension is just a type of spring.
Define a coordinate x as the direction in which a spring compresses. The "spring rate" is a function of $x$, and describes the amount of force with which the spring will tend toward equilibrium at any point of compression or extension away from equilibrium. The steeper the rate function, the more a spring will resist additional movement the further it is moved from equilibrium. For a typical coil spring near equilibrium, the rate function is almost linear. If the rate function is concave up, then the spring has a rising rate; that is, the additional force needed to further compress the spring at each point will increase as the spring goes through its travel. If it is concave down, then the spring has a falling rate, with analogous results. Figure 2.13) shows a graph with each type of rate.

Figure 2.13)


The rate of a bicycle suspension is composed of the internal rate of the shock and the rate inherent in the suspension geometry.

Internal shock rates range from near linear to rising. Coil springs tend to have more linear rates, while air springs tend to have rising rates. All frames may be fitted with a range of shocks, which these days generally have one of two lengths and standard mounts. We will not consider shocks further, since they are not an inherent feature of frame geometry.

The contribution to rate from suspension geometry is determined by the way in which the shock mounts, front and rear wheel axles, and main triangle move relative to one another. The front wheel axle establishes frame orientation to the ground but generally may be neglected, since bottom brackets are almost universally $13^{\prime \prime} \pm .5^{\prime \prime}$ from the ground without rider (given a typical assumption for the fork "Crown to Axle Length" or "CAL"). Thus, the rear wheel and BB largely determine the frame orientation to the ground.

Again, we don't want to spend too much time on this, so we will give an example of the contribution from the relative movement of the shock mounts. Similar consideration must be given to the rear wheel axle relative to the rear shock mount and BB.

Establishing the main triangle as our reference frame, the rear shock mount will travel a circular path around some pivot - the main pivot in the case of a monopivot and the upper frame pivot in the case of a 4-bar (our following statements will apply in both cases).

If the tangent of the rear shock mount points near to the front shock mount as the suspension goes through its travel, then the relative movements of the shock mounts will have a neutral influence on suspension rate (by neutral we mean that, given a linear shock, the suspension rate will remain linear).

Figure 2.14) shows a suspension member moving in the frame of the main triangle. As the suspension compresses, if the rear mount tangent is moving into alignment with the front mount, then the path will increase the rise (decrease the fall) in rate. If it is moving out of alignment with the front mount, then the path will decrease the rise (increase the fall) in rate. This is because for a given angle of rotation, the two shock mounts move towards each other the most when the rear mount tangent is through the front mount.

Figure 2.14)


If we are dealing with a mono-pivot, then the suspension member is the rear triangle and the rear connection will be to the rear axle. If we have a 4-bar, then the suspension member is the upper link and the rear connection will be to the rear link. In both cases, the larger the radius of the rear shock mount path, the larger will be the rate curvature due to geometry. Also, the longer the suspension member; the larger will be the magnification of the internal shock rate curvature, since the wheel will travel a greater distance for a given distance of rear shock travel.

This is most of the ballgame for a mono-pivot (minus only wheel path). For a 4-bar, one must do a similar analysis for the tangent of the upper rear pivot relative to the rear wheel axle. At any position in travel, if the tangent is pointing at the wheel axle, then the shock will compress least for a given amount of wheel travel. In most 4-bars this pivot has a path that will diminish the rate, and again, the larger the path radius of this pivot the larger will be the rate function curvature. The paths of the rear shock mount and upper rear pivot thus define the over all effect in a given 4-bar, minus wheel path.

## Path Analysis.

## Chapter III - Path Analysis.

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## Path Analysis Main Assertions.

Read this section.
This is the central point of the entire work.
This section is moderately difficult.

1) 2) All measures of suspension performance depend almost entirely on the paths of the following specified components relative to any reference frame defined by one of the bicycle frame members: Handlebars, seat, bottom bracket (BB), front and rear wheel axles, shock mounts, and rear brake.

As is explained in the "Reference Frames" section, establishing a frame member as our non-inertial reference does not mean that it will not move. It will translate and rotate, and our reference frame will move with it.

The above "specified" components will always move along paths or onedimensional spaces in the reference frame of one of the supporting bicycle frame members, as a practical matter. The path tangents determine how any bike will behave at any point in time. The path curvatures determine how the bike will behave over time.

If we wish to compare two designs, we should identify a frame member common to both designs. The more alike the paths are in any two suspensions, in the common reference, the closer will be the performance of the frames that produce them. In practice, the bars and seat will always define the best reference and this will be the reference for all analysis in this work (though sometimes it can be interesting to see how paths compare from reference to reference).

Mass and its distribution play an important role in any mechanism. However, the main triangle and rider are usually about 60 times as massive as the suspension members (not including the shock). The movement of rider/main triangle mass will depend on the movements of the main triangle components (seat, bottom bracket, and handlebars), even with a non-integrated main triangle (bottom bracket moving with respect to the bars and seat). In addition, the differences in mass movement of the suspension members between different designs with similar component paths are relatively small. This makes the ground and the rider/main triangle the only two significant masses in bicycle physics.

These mass considerations are what allow for PA. We have covered mass approximations in the "Approximation" section of "Some Important Concepts.". However, when and how to apply approximation can be a very difficult issue, so in the "Mass Approximation" section below, we will explain in detail how mass approximation allows for PA.

Naturally, each individual rider will produce a unique mass distribution. When we say that we can determine suspension performance by the paths, we mean that we can know the performance of the frame for any set of assumptions for relevant physical quantities, such as rider mass distribution or contributions from the suspension fork.

Friction in the suspension mechanism will always act to oppose the movement of components along their paths and will ultimately be directed tangent to the path. Friction magnitude can for the most part be controlled in one type of geometry as well as another. Thus, while we might find one particular suspension bike to have a favorable amount of friction relative to another, friction does not lend any advantage to one type of suspension over another.

Note that the forces between components are critical in determining suspension performance. However, all lines of force, whether they are through the rider, the chain, or external are equally producible in all designs. They thus do not distinguish one design from another. However, it is very helpful to understand how the forces and torques act on and within a bicycle.

Frame stiffness is an important factor in bicycle performance. However, it is much more an issue for handling (a topic not covered in this work), particularly high speed cornering, then anything else. With regard to pedaling, braking, and shock absorption, one only need be wary of the very lightest frames. It has been several years since the author has been aware of any new frames on the market that are so severely under-built as to cause real problems for pedaling, braking, and shock absorption, beyond bad choices and defects in materials and manufacturing, that lead to frame failure (also not covered in this work).

This leaves geometry as the overriding issue in suspension performance regarding pedaling, braking, and shock absorption.

In most cases, the full machinery of PA is not necessary since the paths of components may determine the orientations of their supporting structures (frame members, fork, etc.). For example, the BB and seat may fully determine the main triangle, so one could simply look at that body rather then the attached components.

However, in cases such as the i -Drive, the full machinery of PA is the only practical method of analysis. Analysis of the i-Drive by any other method would be extremely complicated. The power of PA will be revealed in the extreme simplicity of i-Drive analysis using this method.

We will give an analysis of the i-Drive theory, Ellsworth's "Instant Center Tracking" (ICT) theory, and other erroneous theories at the end of this paper.

## P.A. Basics.

## Read this section if:

You want to verify for yourself the validity of the Path Analysis main assertions and understand the details of how and why Path Analysis works.

## Skip this section if:

You will accept the main assertions and are just interested in using Path Analysis to make conclusions about what suspensions can do and comparisons between various bikes.

This section is moderately difficult.
A) A) Mass Approximation.

As stated above:
Path Analysis works because the mass of the rider/main triangle dominates all mass in a bicycle. In addition, the differences in mass movement of the suspension members between different designs with similar component paths are relatively small. This makes the ground and the rider/main triangle the only two significant masses in bicycle physics.

Furthermore, all forces on the rear suspension members, other then those directly between suspension members (through pivots and so forth), are directed through the PA specified components. Since suspension member mass is not significant and the suspension members control motion between the two significant masses, it is enough to consider the forces between the PA specified components.

We now look at this a little more closely.
Consider Figure 3.1 A). Here, we have a main triangle and swing arm attached to a base by a pivot. This is actually the proper model with which to analyze bicycle suspensions, minus some contributions from the front fork. If the force F were calibrated to the force of gravity, you would have most of the situation for the analogous dual suspension bike, in a particular gear (no human rider could really produce such high values, but the values can be reduced by tilting the mechanism backward, into the page). Note that the main triangle can move only in a certain restricted way relative to the base and that the lower pivot, analogous to the rear axle on a bike can move only in a certain path relative to the main triangle.
\{We have drawn the pictures vertically symmetrical and the linkages to form parallelograms. But the model should be taken more generally to include any typically shaped front triangle and lengths of linkage members. The model should also be considered in all reasonable positions.\}

Figure 3.1)


Next, consider Figure 3.1 B). Here, we have a main triangle and a 4-bar linkage attached to a base by a pivot. This produces precisely the same paths as the mechanism in Figure 3.1 A). In fact, if we neglect the masses of the swing arm and linkage, we would have identical situations in both A) and B). Figure 3.1 C) shows both suspensions on the mechanism at once, from which we see that both suspensions will work harmoniously with one another. This foretells an analytical device that I have conceived, called a natural mirror bike, which we will discuss below in "The Natural Mirror Bike." section. Now the question is, "Can we neglect the masses of the suspension members?"

If the mass of the swing arm were very large compared to the mass of the main triangle and the mass of the 4-bar linkage were very small compared to the main triangle, then it is easy to see that PA would not apply. In A), the main triangle would rotate around the upper pivot with relatively little motion from the swing arm when $F$ is applied. In B), the main triangle and linkage would move very differently from A), the linkage moving more drastically then the swing arm, producing a very different physical situation. But this is not the case in a bicycle.

A typical dual suspension frame weighs about five to six pounds, without the rear shock. Of this, the rear might take up 2.5 pounds. Now in bikes for which there is any utility in comparison, the mass difference between any two types of rear suspensions is going to be less then half a pound. The rider/main triangle,
on the other hand, averages about 150 pounds, at least. This leads to a mass difference of less then $0.3 \%$ between the vast majority of mechanisms.

In addition, the movement of mass through suspension compression depends largely on the relative motion of the specified components, within the range of motion for all viable suspension bikes. Considering the movements of the links in a 4-bar linkage, one sees that the overall movement of mass is very similar to the movement of mass in a mono-pivot swing arm (though not exactly the same). The movement of mass in the GT i-Drive is almost identical to a monopivot, the only (insignificant) differences being the movement of the eccentric on the swing arm and of the "dogbone".

As a result, we may neglect the suspension members and focus exclusively on the paths that they produce, as we have drawn in Figure 3.2 A) for the mechanisms in Figure 3.1). Here, we have drawn a circular path for the lower pivot about the main triangle. This contains all of the significant information concerning how the mechanisms will work. Figure 3.2 B) shows the type of motion allowed by all equivalent mechanisms.

Figure 3.2)


We have demonstrated these principles for a 4-bar vs. a mono-pivot, but they apply in general, since the masses of suspension members will always weigh about the same as the examples here.

## B) B) Forces Between Linearly Constrained Particles.

Suspension performance is determined essentially by the relative movements of the PA-specified components. This is because the interfaces to the external world (the wheels) are identical in all bicycles for which there is a utility in comparison. We can thus do our analysis based entirely on these internal workings and neglect any external interactions (with the ground for example). This simplifies matters in that our analysis may involve less degrees of freedom.

As stated above, the specified bicycle components move along paths or onedimensional spaces, assuming a reference frame attached to one of the supporting frame members. How each moves will depend on the sum of forces exerted between it and the other components in the system. So lets get some idea about how to treat objects moving along such paths by looking at some examples.

Suppose you have an $\mathrm{x}, \mathrm{y}$-axis. A particle, such as a marble, is restricted via some mechanism to move freely along the diagonal in the first quadrant. Any number of mechanisms could achieve the restricted movement. Now suppose there was a force F pushing on the particle in the x -direction. The equations of motion for the particle would involve $\mathrm{F} * \cos \left(45^{\circ}\right)$, that is, the direction of motion, but not the mechanism that restricts the degrees of freedom. Figure 3.3) shows the path, force, and force component along the path.

Figure 3.3)


Two particles move along paths relative to one another. If a force is induced between them, it will cause both particles to accelerate along their respective paths in the direction corresponding to the tangent component of the force along the path. Figure 3.4 A ) shows this scenario.

Figure 3.4)


Next consider two wheel axles that are restricted to travel along the same paths relative to one another as the particles in A ), with axles at the same points. The same magnitude of force as in A) is exerted at the wheel edges. Figure 3.4 B ) shows this situation.

Notice that the forces at the axles are in the direction of the force line between the cogs, which is different from the force direction on the particles in A). The forces are also of a different magnitude due to the inertia of the wheels. Particularly important is that these two forces at the axles are not co-linear. The components along the paths in B are in the same directions as those in A , but will generally be of different magnitudes due to both the differences in overall force direction and magnitude at the axles.

Now consider a particle such as a wheel axle that is restricted to travel along a particular path relative to other components in a mechanism (a main triangle for example). If forces are exerted by the other components on this axle, by whatever means, the axle will tend to move in the direction along its path that corresponds to the tangent component of the sum of the forces. The magnitude of the tangent component determines the motion of the axle. Similar considerations exist for the other components.

If one component's path in a mechanism is a function of another's path (in part or whole), then it does not matter how that relation is achieved; any mechanism will produce the same physical results. An example of such a functional relationship is that between a seat, handle bars, and bottom bracket, which define the main triangle. An essentially equivalent functional relationship is produced in GT's i-Drive.

As a result, if we want to consider performance in non-URTS, we need only examine the paths of the wheel axles, shock mounts, and brake, relative to the main triangle.

Figure 3.5) shows a rear axle path in relation to a main triangle for a non-URT. The gray lines denote several possible axle paths. The red line shows a line of force (through the chain). The green arrow shows the force induced at the wheel axle. The blue arrow shows the component of force along the wheel path at the axle.

Figure 3.5)

*** Here is a central point. In the small neighborhood around the axle denoted by the orange lines, the paths are identical. However, above the neighborhood they diverge wildly, one being circular and the others being of more radical curvature. We pictured this to emphasize that the path tangent is what counts at any moment in time. Other aspects of the paths have no bearing on what happens in our small neighborhood around the axle. In little neighborhoods around all points in a path, all suspensions with similar paths in that neighborhood behave similarly; in particular, they behave like some monopivot. When we shrink the neighborhoods to zero, we see that the tangent to the path determines suspension behavior at any point in the path. $* * *$

This might seem strange if we consider mechanisms that produce paths with very different radii of curvature. But remember, it is what happens as the suspensions move away from the particular points on the paths with common tangents that make the situations differ. Greatly differing curvature will produce swiftly diverging physical situations.

The following example should allay concerns about whether or not the tangent can really contain all of the information necessary to evaluate a given situation regardless of path curvature.

Figures 3.6 A ) and B) depict main triangles attached at different points to swing arms of different length. The paths that these swingarms produce are of different radii, but have the same path tangents at the initial locations of the base/swingarm pivots. We depict the mechanisms as being horizontal and viewed from above, so that we may start at equilibrium before a force F is applied to the main triangle. Without loss of generality, we choose the swing arms to be aligned along the $y$-axis.

Figure 3.6)
View from above


The force F may be applied at any point, in any direction. We chose to place it in such a way that the direction extends between the two frame/swingarm pivot locations in the different mechanisms because this is the situation that is most likely to cause concern.

Neglecting swingarm mass, we see that the $x$-component of $F, F_{x}$, will cause essentially identical initial movements of the main triangles in the two mechanisms, since this component is perpendicular to the swing arm. We also see that the $y$-component of $F, F_{y}$, will have the same lever arm about both of the pivots in both cases and thus also will cause identical initial movements of the main triangles. This means that the initial movements of both mechanisms due to the total force will be identical. The two situations will diverge as the swing arm/base pivot paths diverge outside of the initial little neighborhood
around the initial positions. But in the initial positions, the physical situations are identical.

In reality, a swing arm as large as the one in Figure 3.6 A) might weigh a few pounds more then the one in B) (though the difference would still be less then $2 \%$ of the rider/main triangle mass). But such swing arms do not exist in real bikes. 4-bar frame members that can produce a path curvature similar to the one in A) do not weigh substantially more then ordinary mono-pivot swingarms and the mass tends to move in a generally similar fashion over all. So as we observed earlier, neglecting the masses of the suspension components is a good approximation in our analysis.

## The Natural Mirror Bike.

## Read this section.

It is not technically difficult and the "Natural Mirror" conceptual device is the most easily understood confirmation for the validity of Path Analysis.

The best intuitive confirmation that one can have for the validity of Path Analysis is to imagine putting two different suspension mechanisms on one bike simultaneously. There would be no conflict between them as long as the component paths were the same for both mechanisms. Shortly after I first published the "simultaneous suspension", a particular version of my idea was proposed that would have one side of a bike constructed from a mono-pivot and the other from a 4-bar with a circular rear axle path. More generally, we may construct a bike with two different suspension mechanisms on either side, each having the same component paths. I will refer to such a bike as a "natural mirror" or simply "mirror" suspension bike, since the true nature of each suspension is mirrored on the other side.

We can include the paths of all components as part of a natural mirror analysis, or only those for which we may have a particular interest. For example, if we wish to compare only wheel paths, we may imagine pairing up frames with identical wheel paths and it will not matter whether other components, such as the shock mounts, also have identical paths.

In evaluating the validity of a theory, physicists often examine certain "obvious" cases to see if the theory makes sense. Here we examine several designs with circular rear axle paths, to demonstrate that they will all perform identically under pedaling (suspension rate adjustments accounted for in the last example).

Suppose we start on one side of a mirror bike with a 4-bar suspension in which the "bars" determine a parallelogram - that is, the upper arm is equal in length to the lower arm and the forward arm (the main triangle between the two forward pivots) is equal to the rear. We call this a "parallel" 4-bar. The wheel path (both for pivot on the chain stay or seat stay) is circular.

On the other side of the bike, we can use a mono-pivot, with main pivot at the same height above the 4-bar main pivot as the wheel axle is above the 4-bar rear pivot.

We refer to this bike as a "parallel/mono" mirror and both sides produce the same path.

We could even make the shock mounts have equivalent paths by mounting a shock to the mono-pivot in the same way that we mount a shock to one of the horizontal 4-bar arms. Each side of the bike will perform exactly the same as the other. Figure 3.7) shows both of the above suspensions from the side.

Figure 3.7)


Let us now consider another 4-bar. This time though, we will make the two forward pivots coaxial to produce what we call a "pp-coaxial" 4-bar. The pivots will still be physically attached to the main triangle separately and thus the suspension will constitute a true 4-bar. The wheel in this case also has a circular path and thus this suspension can be put in with either of the other two. We will call this 4-bar combined with a mono-pivot a "pp-coaxial/mono" mirror. See Figure 3.8) for this example.

Figure 3.8)


Lastly, let us consider a 4-bar with rear wheel mounted coaxially with the rear lower pivot. It does not matter whether the rear wheel is mounted physically to the chain stay or the seat stay, both will behave the same, as the wheel will have the same path. We will call a mirror bike with this suspension and a monopivot a "wp-coaxial/mono" mirror. The configurations of the 4-bar upper links contribute only to the suspension rate in this case. Adjusting the relative paths of the shock mounts as well as the "internal" rates of the shocks may be done to more or less match the over all suspension rates of the two sides. Figure 3.9) shows this mechanism.

Figure 3.9)


All of these are examples of very different suspension configurations that will behave exactly the same while not under braking (shock tuning accounted in the last case), because the rear axle paths are the same - namely circular. The shock mount paths in the first two examples are not identical in space, but are identical in relative motion and so cause no conflict.

## Paths and Performance.

## Read this section.

This section explains the important considerations involved in most of the full suspension frames built today.

This section is less difficult, except in one or two places perhaps, and is of great use to consumers.
\{A technical note about the pictures in this segment: The main triangles are not drawn to scale and the paths are not meant to represent solutions for any particular real-world situations or as endorsements for any particular designs they are constructed merely to illustrate the points.\}

Before analyzing paths, we make a few general comments on some other issues.
Both major suspension types (mono-pivots and 4-bars) may be as light or as strong as any dual suspension bike can viably be, examples of both having found success in XC and DH. Both types can also achieve comparable lateral stiffness for a given frame mass.

Mono-pivots are a bit simpler of design, but most of today's 4-bars are about as reliable.

Some 4-bars offer adjustable travel and geometry. This is equally possible with a mono-pivot, but as of this writing, mono-pivot manufacturers have yet to answer in a substantial way.
A) A) Mono-Pivot and 4-Bar Rear Axle Paths.

4-bar rear axle paths can deviate from those of conventional mono-pivots in three different ways:

First, mono-pivot paths will always be circular about the main pivot. 4-bars can have varying path curvature. The easiest way to see this is to consider Figure 3.10 A). Draw the complete circular path of the upper rear pivot as the upper link rotates about the upper front pivot. Next, consider the path of the lower rear pivot, as the upper rear pivot goes through its revolutions. This lower pivot will move back and forth along a circular arc. By "'Nature Varies Smoothly' (NVS).", we see that the paths of points along the rear link will be much like the upper link circle for points close to the upper link, and much like the lower link
arc for points close to the lower link. The paths will gradually change from one to the other as the points vary along the rear link. An axle mounted somewhere in between will have varying path curvature.

Figure 3.10)



At the moment, the "The Virtual Pivot Point (VPP)." concept, conceived by Outland and soon to be re-introduced by Santa Cruz and Intense, is unique among viable concepts in its capability to produce significantly variable curvature. However, as we will see, current examples do not take any real advantage of the possibilities.

Secondly, conventional mono-pivots do not have main pivots located within the wheel radius. This limits the minimum radius of axle path curvature to at least the size of the rear wheel.

A mono-pivot could achieve a tighter curvature only if the pivot were split into right and left. We have proposed such a bike, which we call the "split-pivot mono". This design is in fact viable and will have the added benefit of a more stable pivot. Figure 3.10 C ) shows the tight curvature. We thus do not consider tight curvature to be an inherent advantage of 4-bars over monopivots, since the split-pivot mono can achieve the path. Although no such bikes are in current production, the split-pivot mono was the motivation behind Cannondale's new Scalpel.

4-bars can achieve a tightly curved path centered inside the rear wheel radius. Figures 3.10 A ) and B) show a 4-bar with the same tangent as our split-pivot
mono both at equilibrium and compression. We have achieved our example by having the IC move backward as it moves down. This is essentially the Giant NRS design. The Rocky Mountain ETS-X70 also uses a linkage system to create a center of curvature inside the wheel radius.

And thirdly, mono-pivots will always have pivots located within the body of the bicycle frame. 4-bars can achieve a more widely curved path, as is the case in the current "Virtual Pivot Point (VPP)." designs.
B) B) Shock Absorption ("coasting" situations).

We handle only coasting situations here, since suspension issues related to pedaling and braking will be handled specifically in those sections.

A bicycle suspension may be suddenly compressed by the ground either through wheel contact with an obstacle such as a rock or from the impact of a drop-off. In general, we believe that a widely curved rear axle path running slightly up and back is the best solution. Tight curves, either circular or varying are generally inferior for shock absorption. However, this deficiency may be mitigated to some degree by having the path tangent tilting backward through all or most of travel (for example, having a high main pivot, either real or virtual), as is the case in the The Rocky Mountain ETS-X70 and, substantially, the Giant NRS. One might also find that short travel designs such as the Cannondale Scalpel do not have enough travel for this deficiency to be significant.

In the case of a drop-off, the situation is obvious. A linear path will offer the smoothest, most consistent compliance.

In the case of an obstacle, the bump force will be up and back relative to the frame, so the initial tangent should be up and back. The direction of the force will turn more vertical as the bike clears objects of "ride-able" size, so a widely curving path turning slightly upward should be ideal. Experiment should determine the path incline and radius of curvature that produces the best result on average.

Rising rates benefit short travel designs, since this will allow better initial compliance, while reducing the probability of hard bottom-outs.
C) C) Pedaling (non-URTs only).

Non-URT generally means BB on the front triangle. These bikes dominate the industry these days and most are either mono-pivots or 4-bars. Here we examine pedaling of non-URTs, asking specifically, "Are there any relative merits between the mono-pivot and 4-bar design concepts under pedaling, and if
so, what are the considerations involved?" In asking this fundamental and rather popular question, we will get a good general idea of what attributes really effect non-URT pedaling performance.

We observed in Figure 3.5) of the "Forces Between Linearly Constrained Particles." section that the component path tangents determine how any suspension will perform at any point in time.

This means that, neglecting friction in the mechanism, each particular geometry will have its maximum effectiveness only in certain "ideal" gears (from a practical standpoint, this could mean one gear or several). Any others sets of gears will produce different forces on the mechanism, leading to different components of force along the tangents. The further the gearing from ideal, the more reactive any suspension geometry will be.

For a given deviation away from ideal gearing, "suspension rate" (spring stiffening) will determine the amount of reaction from a pedal stroke. Shorter travel suspensions tend to be less reactive to pedaling then longer travel versions, since short travel designs should have higher more rising rates (in part due to the fact that many use air shocks these days). In practice, the actual rates in the shallow regions of travel where pedaling will be affected will largely be a function of the total travel length of the rear wheel path.

Most frames mate well with their stock shocks, and all common suspension types can achieve the really useful rates (linear or rising). So rate is only a real issue for those wishing to swap different coil and air shocks in and out of a given frame.

Since rate in the shallow regions of travel will largely be a function of total wheel path length and is of secondary importance to most people, we will not further consider rate here. We refer those still interested, to the "Suspension Rate." section in chapter II.

Any comment on frame performance must be made with respect to a range of forks, just as is the case with rider mass. So an assumption must be made for fork characteristics. In addition, dual suspension bottom brackets (BBs) are almost universally $13^{\prime \prime} \pm .5^{\prime \prime}$ from the ground without rider, given a typical assumption for the fork "Crown to Axle Length" or "CAL". Thus, the rear wheel and BB largely determine the frame orientation to the ground. So, after noting the required fork assumption, we can neglect the front wheel without much problem. [If one is uncomfortable with this, then one may certainly consider the front wheel axle path. This and the rear axle path will determine orientation of the main triangle to the ground (again, a CAL assumption must be made)].

## We see then that the pedaling performance of any non-URT will be determined largely by the rear axle path (including the length, which will give us a good idea of the rate influence).

So PA can become very simplified for certain types of frames and certain types of analyses. This simplified version of PA has been known for some time and been used by numerous designers in the past.

Given that any sort of design can produce all possible rear axle tangents, potential differences between the two non-URT types, and between the various individual non-URT designs in general, will have to come from differing possibilities as the rear axles move through their paths. So we now examine whether or not viable varying curvature, tight curvature, and/or wide curvature offer significant performance advantages or drawbacks.

First, let us consider what might be an ideal path to minimize suspension reaction to pedaling.

For ease of discussion, we will assume 1-1 gearing. With this gearing, there will be no feedback to the pedals as the suspension goes through its travel, if we have a circular path centered at the BB . That is, the distance from the BB to the rear axle must be constant, as in a mono-pivot with main pivot coaxial to the BB. (If the gearing is larger, then the distance must increase to eliminate feedback, while the opposite is true for smaller gearing.) Figure 3.11 A ) shows this wheel path:

Figure 3.11)


Upper Circular wheel Path siogmont Esontored it HSt Negative Sloping Wheel Hall Sexmoni


Continuing with 1-1: To counter squat and some compressive chain effect at equilibrium, the path tangent must have a negative slope (be tilted back counterclockwise from vertical). This will counter squat with chain force and by altering the effect of bike acceleration on the swing arm. Figure 3.11 B ) shows this situation, retaining the overall circular wheel path. But now we have a situation with some bump feedback to the pedals.

The significance of feedback is very debated. It certainly achieves its greatest significance away from ideal gearing, usually in the smaller gears where chainlength growth (between the two cogs) is increased and the effect of smooth wheel spin disruption on the crank will be magnified by a large "reverse gearing". However, we want to be very clear on one point. There is no free lunch - to any degree that you have anti-squat, you will also have some degree of bump feedback, regardless of what certain manufacturers claim.

The vast majority of experienced riders give great importance to a smooth pedal stroke, so feedback is generally seen as something to be minimized if possible. Figure 3.11 C) shows a type of path that would allow the chain to counter squat at equilibrium, while limiting bump feedback to the pedals. This path would have a small segment around equilibrium of just the right negative slope, with BB centered circular segments above and below. Most would consider this the ideal situation.

In principle, a 4-bar can also achieve a path similar to the stated ideal by having a progressively "tightening curvature" as the suspension compresses through its travel. The subtle features of the Figure 3.11 C) path would be lost, but the broader shape would not be grossly different. This would allow the suspension to control squat at equilibrium, while providing less feedback then a conventional radius circular path, both above and below (below being a lesser issue, since most of the travel is above).

So we see that certain variable curvature paths can offer an advantage with regard to pedaling, in principle.
"The Virtual Pivot Point (VPP)." design concepts are capable of producing an "S-shaped" path somewhat similar to the region around equilibrium for the path in Figure 3.11 C ). These designs would obviously also be capable of producing "tightening curvature" paths. To date, the Outland designs are the only bikes we know of claiming significantly varying curvature. Again, unfortunately, the current examples do not take any real advantage of these possibilities.

A tightly curved circular path above equilibrium can provide an anti-squat path tangent, while curving up more sharply to reduce feedback during compression deep into travel. Such a design should be run with little or no sag, since kick-
forward during suspension extension may become an issue. The Giant NRS, the The Rocky Mountain ETS-X70, the Cannondale Scalpel, and the "split-pivot mono" described above (not in production) are examples of tight curvature designs (though one might find that the ETS-X70 does not have a small enough radius, nor the Scalpel enough travel for this to be significant for him or her).

Wide paths would offer no advantages with regard to pedaling, since they offer no special path tangents and do not address the issue of anti-squat verses feedback.

There may be those who desire anti-squat throughout the travel and consider feedback an acceptable price to pay. For these people, wide curves may offer a perceived advantage under pedaling. However, we feel that this is not a wise position. During large compressions from an obstacle, such as a rock, the rider will be kicked forward when the rear tire encounters the obstacle; so squat is not the issue during this type of suspension compression. The impact of a drop-off will compress the suspension regardless and the rider is likely to be standing (thus creating a completely different pedaling situation from that for which any bike will be designed), so again, squat is not an issue.
D) D) Compromises.

We have seen that rearward axle path tangents at equilibrium should offer some advantage while pedaling over smooth terrain and during shock absorption while coasting. However, this will also produce bump feedback while pedaling over bumps. So we have a tradeoff. Many riders say that they are very sensitive to this trade off, even to the point where differences of less then an inch in main pivot locations are noticeable. Some prefer the generally efficient rearward tangents, while others want the smoother pedaling, more vertical tangents. So we have a compromise with which to deal.

We have also noted that tight curves above equilibrium, whether circular or varying, may help with reducing the bump feedback of a rearward tangent. However, curves tight enough to make a significant difference in the shallow regions of travel where riders are likely to be pedaling may produce inferior bump performance deeper into the travel, since wide curvature should be best for shock absorption. Though again, designs with rearward paths through travel, such as the The Rocky Mountain ETS-X70 and, for the most part, the Giant NRS, may mitigate this compromise to one degree or another.

Furthermore, while variable curvature has its allure, in practice it requires closely spaced pivots near the frame center. As a result, links and pivots in the highly stressed bottom bracket area must be more heavily built. This leads to bigger tradeoffs between weight and durability then in conventional 4 -bars. So variable curvature designs are not without their compromises.

This furthers a theme that we have revisited throughout this work - there are no "right paths" or "right pivot points". We have seen this in mass distribution considerations of having riders with different body types. We have seen this in the fact that no geometry can be completely non-reactive through a pedal stroke, without the help of friction. And now we see it again in the fact that there are choices that must be made, depending on what type of suspension performance one wants.

Human beings can be surprisingly sensitive to physical situations. This author finds that a difference of just two millimeters in the height of a road bike seat makes for the feel of a completely different bike. So we are not surprised to find that some people hold small geometric differences as important and we must assume these positions to be legitimate.

However, we must note that some people experience "have-it-all" performance in some designs from manufacturers that claim such performance (though this is certainly not the case with most experienced riders that this author has encountered). Since we have seen that have-it-all performance is impossible, we must conclude that either the powerful psychosomatic phenomenon is at work or that some of the considerations that we have been exploring are not all that discernable to some people, or perhaps it is a little of both.

All of this makes the question of suspension performance largely philosophical. So to continue another theme, we again suggest that test riding be done, even if it is just a parking lot test, to determine what performance characteristics are right or even discernable for each person.

In the final analysis, none of the major suspension types has a clear advantage over the others. There are lots of happy mono-pivot owners out there (including those with mono-pivot-like Ventanas and Rockies) and there are lots of happy 4-bar owners out there. This pretty much says it all.
E) E) Braking.

We first say a word on floating disc brakes.
Floating disc brakes are rear disc brake mechanisms wherein the brake is mounted on its own linkage arms, which are not part of the load bearing, rear suspension components. Figures 3.12 A and B ) show simple diagrams of this type of mechanism, pictured in red, both at extension and compression.

Figure 3.12)


Many claims are made for floating brakes (see the "False Claims for Floating Brakes." section for an expose on some widely believed things that floating brakes do not do). However, the only thing that these mechanisms do is to give a bike the braking character of the brake mechanism's linkage system. For example, these mechanisms can give a mono-pivot the braking character of a 4bar with suspension geometry identical to the brake mechanism's linkage geometry. This means that mono-pivots with floating brakes may develop some propensity to extend under braking, as is the case with typical 4-bars. 4-bars may or may not develop a change in character under braking, depending on whether or not their suspension geometry is significantly different from that of the floating brake.

Below we will do some analysis involving 4-bar suspensions. Since there is no real distinction under braking between a 4-bar suspension linkage and a floating brake linkage with the same geometry, all statements below regarding 4-bars will also apply to bikes equipped with floating rear brakes.

The biggest question regarding braking in dual suspension bikes is whether or not 4-bars rear-brake better then mono-pivots, in general. We will take up specific theories regarding this question in the "Brake Induced Shock Lockout' (BISL)." section of Chapter V. Here we will examine how the performance of possible 4-bar link configurations compare to those of mono-pivots with identical main pivot locations.

Figure 3.13), depicts a 4-bar suspension frame with various possible locations for the upper, forward pivot, giving an IC at the main pivot. Path Analysis tells us that this frame will rear-brake identically to a mono-pivot with identical main pivot location, at the depicted point in travel, because the path tangents of the rear brakes will be the same in both mechanisms.

Figure 3.13)


This is most clearly seen when considering the two suspensions as part of a mirror bike. In a small neighborhood around the depicted point in travel, the paths of the components will be essentially the same, the 4-bar swingarm and rear link moving just as the mirrored segments of the mono-pivot rear triangle. There is essentially no relative movement about the 4-bar lower rear pivot in the small neighborhood about this position in travel, so it does not matter whether or not the lower rear pivot is even there.

We have drawn Figure 3.13) with a 90 deg. angle between the rear and upper links in order to produce the most visually convincing physical situation. But any 4 -bar configuration with IC at the main pivot will brake equivalently to a mono-pivot. To see this, consider Figure 3.14). Here we depict a 4-bar suspension with multiple possibilities for an upper link configuration, attached to a base, and oriented horizontally. All forces in both mono-pivots and 4-bars are identical under braking with the exception of the interplay between the rear wheel, rear brake, and suspension components. Mounting the frame horizontally allows us to consider this interplay in isolation.

Figure 3.14)


Under braking, a force F is induced from the rear wheel through the brake to the rear link. Neglecting the mass of the upper link, which is very small, we see that this force will in turn be transmitted to the upper link and ultimately to the main triangle, directly down the axis of the upper link.

To see this, it may help to consider the forces involved between the main triangle and suspension components of the 4-bar as we did the force in Figure 3.6. Decompose the force through the upper rear pivot, from the rear link to the upper link, into forces parallel and perpendicular to the upper link. Do the same for the force between the upper link and the frame.

We see that the torque balance about the main pivot in a 4-bar with IC at the main pivot will be the same as in an equivalently main pivoted mono-pivot. We also see that an IC in front of the main pivot will create a suspension more extending then a mono-pivot under braking (this is sometimes called "brakejack"), since the rear brake path ascends through suspension travel more then it would in a mono-pivot with identical main pivot location. An IC behind the main pivot will create a suspension that is more compressing.

Note: It is a very common misconception [see the braking analysis of Ellsworth's "Instant Center Tracking" (ICT)] to believe that the angle between the rear and upper links is what determines the brake's effect on the suspension.

But it is the force transferred to the rider/main triangle that ultimately determines whether or not the suspension will react.

Imagine varying the angle between the rear and upper links, while holding the axes of the upper and lower links constant, producing a constant IC location under variation. The components of the force on the upper and lower links, from the rear link, are changing, but so too are the lever arms. In the end, this variation in angle will not change the brake's effect on the suspension.

We have done numerous experiments on mono-pivots that show them to be generally neutral (neither extending nor compressing) under braking [see the "'Brake Induced Shock Lockout' (BISL)." section]. For both mono-pivots and IC/main pivot coaxial 4-bars then, the effects on the main triangle will remain largely the same throughout a smooth-surface braking process.

Most 4-bars have an IC in front of the main pivot, causing them to extend relative to most mono-pivots, under smooth-surface braking. Extension has been confirmed by experiments on an Intense Tracer, a very typical 4-bar design. Interestingly, this extension has the potential to cause the suspension to press against the top-out bumper in very short travel designs meant to be run with little or no sag. This would be especially true in designs such as the Giant NRS. A bump force would have to overcome the extending brake force before the suspension would compress.

Some 4-bars, such as the Jamis Dakars and the Psycle Werks Wild Hare, with ICs just about at the main pivot, will brake equivalently to mono-pivots on a smooth surface.

The Yeti AS-R, which has an IC behind the main pivot, will be compressive under smooth-surface braking relative to mono-pivots.

When a 4-bar hits a bump and compresses, the instant center will move, thus changing the geometrically inherent suspension rate under braking. If the upper link in Figure 3.13) points up from the main triangle (rotates clockwise under compression), then the contribution to the effective suspension rate from the wheel force on the brake will become less extending/more compressing as the suspension compresses over a bump. Similarly, if the upper link in Figure 3.13) points down (rotates counterclockwise under compression), then the opposite will be true.

This may offset to some degree the tendency of most 4-bars to become more extensive with application of the rear brake.

The effect will be smallest for bikes with upper and rear links starting out at 90 deg. to each other, such as in the Dakars and Wild Hare. So these bikes should still brake almost exactly like mono-pivots, regardless of the ground conditions.

I hope that you all have found this work useful and enjoyable. I wish you happy trails.

Ken Sasaki.


#### Abstract

Path Analysis. Chapter IV - Wheel Path Analyses of Some Existing Models.

Theory, text, illustrations, and editing by Ken Sasaki. 4-bar path analysis by Peter Ejvinsson. Spanish Version translated by Antonio Osuna. "Linkage" suspension simulation by Gergely Kovacs. © Kenneth M. Sasaki 2001, all rights reserved. \{The authors welcome the reposting or reprinting of this page or any part of it, so long as full credit is given to the authors.\}


## Read these sections.

They are not technically difficult and demonstrate how most of the major 4-bar designs will perform relative to one another under power.

The following CAD analyses of various 4-bar rear axle paths were done by Peter Ejvinsson. These analyses show the distances of the various rear axles from fixed points as the suspensions go through their travels. Note that with the exception of the Virtual Pivot Point (VPP), these distances are remarkably constant - that is, the wheel paths are almost circular within the range of motion. We thus say that these axles have "virtual pivots". Note that other points on the rear links will have other virtual pivots, or none at all.

This means that, with the noted exception, the following suspensions could be paired extremely well with mono-pivots (a "split-pivot mono" would be needed in some cases), in natural mirror bikes, and will behave almost exactly like mono-pivots under pedaling and non-braking shock absorption (since all nonURTs can achieve all of the really useful suspension rates).

These frames encompass the current major design configurations used in the vast majority of chain stay pivot bikes (exceptions include the Lawwill parallellink suspensions which are even more circular, some designs with pivots closely spaced near the center of the frame such as the Schwinn Rocket 88, and the new Maverick, which we have yet to evaluate).

We have not plotted mono-pivot or seat stay pivot bikes, since their circular nature should be trivial to evaluate.

## Typical Horst Link Designs.

"Horst link" designs refer to frames with lower rear pivots mounted on the chain stays, forward and below the height of the rear axle.

We have not bothered to plot the Ellsworth bikes, since these will be even more circular then the designs pictured.

There has been a persistent myth circulating that chainstay pivot suspensions "isolate" forces on the rear link and thus are not affected by pedaling or braking. But as has been noted in the "Internal Force' Theories." section, this is entirely false.

Typical Horst link designs such as the ones below are all very circular and will perform identically to the analogous mono-pivots under pedaling. Under braking, the pictured suspensions will have a tendency to extend. The virtual pivots on some, such as the Tracer, are a bit farther back then is the case in any standard mono-pivots, but the virtual pivots are not within the rear wheel radius.

Figure 4.1)
The Specialized FSR:

Figure 4.2)
The Intense


Figure 4.3)
The Titus Switchblade:

Figure 4.4)
The Turner


The Giant NRS (Linkage data).
Figure 4.5)
Figure 4.6)


Figures 4.5) depicts the virtual pivot, while Figure 4.6) shows the centers of curvature and lines perpendicular to the wheel path for the Giant NRS.

Note that the "virtual pivot" on the NRS is well within the rear wheel radius and relatively high above the bottom bracket. This design will behave very much like a "split-pivot mono", described above. The idea behind this suspension, as explained on the Giant web site, is to have the suspension run just at no sag, with the rider's weight exactly countering the force from the shock. The wheel path is tilted well back relative to the frame, so that the chain force will want to extend the suspension. In this way, the NRS eliminates suspension activation
through pedaling. The effect of a pedal stroke on the suspension is like a momentary increase in force from the shock. Force from a bump must overcome this additional force before the shock will activate during pedaling.

The small radius of the wheel path reduces what will probably be considerable bump feedback. These bikes should accelerate very well, but will probably not handle pedaling through technical obstacles as well as lower-pivot designs.

Tight curvature generally reduces suspension performance over large bumps to some degree, however, in the NRS, this is largely mitigated due to the high and rearward virtual pivot.

In addition, the configuration of links will probably induce some locking effect of the suspension, under braking (see the "Braking." section for a full analysis).

We have no idea of precisely how Giant arrived at their geometry, or if the ideas behind any quantitative force theory they are using are completely sound. But as far as the information that they do provide goes, there is no overt error.

## The Rocky Mountain ETS-X70 (Linkage data).

Figure 4.7)


Figure 4.7) shows the paths for the rear axle, IC, and center of curvature, as well as the lines perpendicular to the rear axle path for the Rocky Mountain ETSX70.

There is a fairly well defined virtual pivot located well above and behind the bottom bracket. The height is similar to that of the NRS, but farther forward. This will give the bike characteristics similar to that of a very high pivot monopivot; though the virtual pivot is located a bit farther back then could be the case on a standard mono-pivot. One should find quite a bit of both anti-squat and kickback with this frame, though the tradeoff will be a tad less then in conventional mono-pivots. Shock absorption should be good, in spite of the relatively tight wheel path curvature, due to the high virtual pivot and rearward sloping wheel path through travel.

Having the links located above the chain line is a nice feature, however, this does raise concerns regarding lateral stiffness. Preliminary observations for frame flex, taken by pressing laterally on a pedal located at six o'clock, seem to indicate that the bike has lateral stiffness on par with typical XC dualies. The author has done some accelerations on this bike and found that the frame has
adequate stiffness with regard to pedaling. Perhaps a greater concern, however, is high-speed cornering, above say 25 to 30 mph . To date, we have no observations on how the frame reacts under these conditions.

This is a very unique frame design, which will probably prove well suited for some, but not for others. As always, we say that test rides should always be taken.

## The Cannondale Scalpel.

The Scalpel is another tight-radius design. The bike was inspired by a "splitpivot mono" prototype, which can be viewed on the Cannondale web site.

Figure 4.8) shows a picture of the rear suspension, taken from Cannondale's web site (with permission). The thin, flexible section of the chain stays is meant to act like a pivot. The path tangent starts out at nearly vertical and curves forward. The chain stay length is initially increasing, since the BB is centered below the height of the rear axle. We have not bothered to plot the path, since the center of curvature is going to be at the thin part of the stays. The intended benefits, as explained on the Cannondale web site, are an increasing effective chain stay at sag, with a tight curvature to reduce bump feedback.

Figure 4.8)


There is not much more to say here, since the concepts are fairly straightforward. The one thing to add is that chain stay bending does not have to be localized, as it is in the Scalpel, to produce a rear axle path radius of curvature smaller then the wheel radius. However, there is a potential advantage in localizing the bend in that the exact curvature can be more precisely controlled.

The location of the bend in the Scalpel probably produces a radius of curvature similar to typical "soft-tail" designs. The initial path tangent is tilted just slightly more rearward, relative to typical soft-tails, since the bend is centered at the thin part of the stays near the top of what are otherwise rather thick stays.

The extremely short travel of most soft-tail designs makes the above path considerations essentially irrelevant. However, in the Scalpel, there may be just enough travel for the tight wheel path to make a difference in bump feedback. The limited travel means that the Scalpel will essentially have no big hit performance, so tight curvature is preempted as a drawback. As always, each rider should test ride, to determine for himself or herself, whether or not any of the design considerations are significant.

## The Virtual Pivot Point.

Starting on September 10, 1996, a series of patents was granted for bicycle suspensions with "S-shaped" paths similar to the region around equilibrium for the path in Figure 3.11 C ] [U.S. patent $5,553,881, \underline{U}$.S. patent $5,628,524$, U.S. patent $5,867,906$, and U.S. patent $6,206,397]$.

The original design, essentially a 4-bar with reinforcing upper links, was produced under the name "Outland". The bikes are no longer in production due to serious errors in application - the frame members and pivots were severely under-built.

A second Outland design, covered in the last patent, will shortly be introduced by Santa Cruz and Intense. This design also has the potential to produce an "Sshaped" path.

By following the "images" links from the patent links above, one may view TIFF images associated with the patents. If your browser does not have TIFF capability, then the "alternatiff" program may be installed to give such a capability. In the following explanation of the VPP concept, we have included what we consider to be the most relevant pictures.

We will now give an explanation of the VPP concept, with an explanation of the original mechanism and some important commentary. We will then do an analysis of the current design configurations being developed by Santa Cruz and Intense.

Figures 4.9) through 4.17) were taken from U.S. patent $6,206,397$.
The following explanation for the VPP concept comes from the abstract of the latest patent:
"A rear suspension system for a bicycle. The system directs the rear wheel along a predetermined, $S$-shaped path as the suspension is compressed. The path is configured to provide a chainstay lengthening effect only at those points where this is needed to counterbalance the pedal inputs of the rider; at those points in the wheel travel path where there is a chainstay lengthening effect, the chain tension which results from the pedal inputs exerts a downward force on the rear wheel, preventing unwanted compression of the suspension. The system employs a dual eccentric crank mechanism mounted adjacent the bottom bracket shell to provide the desired control characteristics."

The intent of this system is similar to that of the path we explored in Figure 3.11 C), which is to provide anti-squat during pedaling through a rearward tilting
path near sag, while reducing the effects of bump feedback by turning the curve back toward a more constant radius about the bottom bracket, away from sag. Figure 4.9) shows the original VPP mechanism, at sag, with path pictured.

Figure 4.9)


Note that the axle is near the bottom of the region where the path takes a backward turn.

Being currently unique in its ability to produce significantly variable curvature, the VPP concept is probably the most intriguing design out at the present time.

The VPP concept will really prove a significant departure from prior designs if manifestations can strike the right balance in having a curve above equilibrium tight enough to reduce bump feedback significantly, but not so tight that suspension performance is compromised through the travel range. This must also be done while maintaining reasonable weight, strength, and durability.

Figure 4.10) shows several possible S-shaped paths in relation to circular paths. The VPP paths here show an obvious deviation from circular.

Figure 4.10)


Figure 4.11) shows an s-shaped path, with lines of chain force that the inventors envisioned as acting on the axle at various points in the path.

Figure 4.11)


Figure 4.12) shows the closely spaced rotating pivots responsible for the Sshaped path.

Figure 4.12)


Figure 4.13) shows the relative orientations of the rotating pivots, as the suspension goes through its travel. Note that the "Instant Center" starts out very low around full extension, as in Figure 4.13 A), giving a fairly vertical path in this region. The "Instant Center" moves higher as the suspension moves toward the central part of travel, as in Figure 4.13 B), giving a rearward tilting path. The "Instant Center" moves back to a low position as the suspension continues on toward full compression, as in Figure 2.23 C), turning the path back forward to reduce effective chainstay lengthening. A very interesting mechanism; this.

Figure 4.13)




It will prove instructive to note how the inventors saw their mechanism in the context of prior designs. The "BACKGROUND OF THE INVENTION" section of U.S. patent $6,206,397$ contains the following:
"Shock absorbing rear suspensions for bicycles are known. In general, however, these have not proven entirely satisfactory in practice.

In most rear suspension assemblies, the rear axle pivots about a single point when subjected to bump forces, as when traversing rough terrain. In these designs, the pedaling forces which are exerted by the rider tend to either compress or extend the spring/damper assembly of the rear suspension. In this respect, the spring/damper assembly of the rear suspension is affected by the pedal force and some of the rider's energy is needlessly wasted.

This effect manifests itself by the common tendency of rear suspension systems to either lock up or 'squat' when the rider pedals. Since most of these systems have a single lever arm which pivots about a single axis, the lock up or squat generally occurs as a result of chain tension acting on the single lever arm. If the single pivot line is above the chain line, the suspension will typically lock up and/or 'jack', thereby providing compliance only when the shock or bump force exceeds the chain tension. Conversely, if the single pivot point of the suspension system is below the chain line, the system will typically squat, since
the chain tension is acting to compress the spring/damper assembly of the rear suspension system, similar to a shock or bump force."

There are several incorrect assertions here:
The first is that a mono-pivot will either "lock up or 'squat' when the rider pedals." Here they have obviously ignored the fact that a conventional monopivot path tangent may be such that there is the minimum possible suspension reaction to pedaling, at equilibrium, for a given gearing.

But most striking is that the inventors have fallen pray to that scourge of the bike industry, "Pivot at the Chain Line' (PCL).", as is amply demonstrated in the last paragraph. This means that they did not properly appreciate front triangle dynamics and were not aware of the ramifications from the chain force running through the wheel, rather then acting directly on the swing arm, as was covered in the ""Center of Mass" (CM)." section.

This adherence to "'Pivot at the Chain Line' (PCL)." also explains the neglect of gearing, mass, and other issues important to bicycle physics, in the patents, the importance of which was demonstrated in "An Intuitive Look at Forces and Torques."

Surprisingly, while the inventors had a very simplistic and incorrect view of bicycle suspension physics, they nevertheless came up with a very interesting mechanism, which has the potential to reduce the dilemma of anti-squat verses bump feedback for a bike run with sag. So, while strictly speaking, the VPP theory could be put in the "Flawed Theories and Bogus Marketing." chapter, we have instead presented the material here, since the error constitutes only a small portion of the ideas involved in a potentially beneficial concept.

We now turn to an analysis of the VPP manifestations currently being developed by Santa Cruz and Intense.

Here is a VPP diagram, in pdf format, that was released by Santa Cruz and Intense. Note that the path depicted is exaggerated (information they unfortunately deleted in the initial magazine release of this picture).

Figures 4.14 A and B ) are diagrams of the new design included in U.S. patent 6,206,397.

Figure 4.14)


Disappointingly, we could discover no paths depicted for this mechanism as part of U.S. patent 6,206,397.

## The Santa Cruz Blur (Linkage data):

Peter Ejvinsson has created the following beautiful cad drawings, showing the most important information about the Santa Cruz Blur.

The first of these drawings, Figure 4.15), depicts the wheel axle path and the IC path (one inch intervals in wheel travel marked) as the suspension moves through its travel. The IC starts out near the BB, initially arcs up and forward, and finishes by continuing forward but slightly down. The wheel path does have a slight " S " shape, which is a bit difficult to see in this picture.

Figure 4.15)


Figure 4.16) depicts the wheel axle path and the path of the center of curvature, as the suspension moves through its travel. The center of curvature starts out behind the bike and quickly moves to negative infinity as the wheel path straightens. As the curvature inverts, the center of curvature jumps to positive infinity, before moving back to a final position well above and slightly behind the BB.

Figure 4.16)


From the behavior of the center of curvature, we see that the path is S-shaped, but only slightly. When the wheel is above one inch into travel, the normal sag for a bike with this amount of travel, the radius of curvature is always very large, until the very end, when the center of curvature moves to a horizontal position common in many of today's mono-pivots. It is for each person to determine whether or not this curvature offers any significant advantage over more conventional designs. In our estimation, the wide curvature should offer good coasting bump performance, however, we see no significant advantages for pedaling in this sort of curve.

Figure 4.17) shows the IC path, the center of curvature path, and the lines perpendicular to the path, at one-inch intervals of wheel travel. Note that the perpendicular lines pass through both the IC and center of curvature positions.

Figure 4.17)


The slopes of the perpendicular lines again show that the wheel path has a slight S-curve. But more importantly, they show us that this bike will perform under pedaling much like a very high-pivot, mono-pivot bike when the suspension is above one inch of travel, again, the typical sag point. This means that the Blur should have pedaling characteristics similar to those well-known in the Santa Cruz Heckler, but even more so, since the path is wider and slopes back even more. That is, there will be more anti-squat then in a Heckler and correspondingly, more bump feedback.

Recently, this author was able to take a short ride on an Intense VPP crosscountry prototype, which has a geometry very similar to that of the Santa Cruz Blur. That ride confirmed the theoretical findings above. The suspension extended under pedaling in all small and most middle ring gears, just as would be the case in a very high-pivot mono-pivot.

Again, it is for each person to determine whether or not this combination of characteristics is right for them. However, it is clear that this bike is subject to the same compromises as designs that have come before.

Those wanting a very high-pivot, hyper-Heckler type of ride will like this bike. We also believe strongly that those who are truly sensitive to bump feedback will not like the bike.

The Santa Cruz V10 (Linkage data):
Figure 4.18) shows the important information for the Santa Cruz V10. The outline of the frame shows the position at full extension. The rear axle path is shown in green, the center of curvature path is shown in turquoise, and the IC path is shown in red, while the lines perpendicular to the path are light orange. Positions in travel are circled at one-inch intervals. The range of travel goes from -2.75 inches to +10 inches of travel, with 0 inches being at full extension. We have plotted the paths beyond full extension in order to show the suspension position needed to produce the S -shaped rear axle curve.

Figure 4.18)


Within the range of travel, the rear axle path does not achieve an S-shape. Rather, it has a wide radius of curvature until the very end of travel, with the path tangents starting out similarly to those of a relatively low-pivot mono-pivot and ending with tangents more similar to higher-pivot designs.

The relatively wide radius of curvature should give the bike good big hit shock absorption, with very little bump kickback. Suspension activation under pedaling should be similar to more conventional medium-height pivot designs on the market.

While we again find that there is no advantage in the tradeoff between antisquat and kickback, and the rear axle path does not achieve the S -shape, within
the range of travel, we nevertheless believe that this design should perform well in its intended downhill application, due to good big bump performance.

The durability and reliability of this frame are unknown, as it is very new at the time of this writing.

## Additional Linkage Data.

Linkage data exist for the frames listed below. We have done no analyses of these frames in this chapter, but we are confident that anyone reaching this section of the work, and having read the major sections of what has come before, should have no trouble in discerning exactly what they want to know about each frame, using Gergely's most excellent Linkage program.

Cannondale Super V. GT lts. GT i-Drive.
HI-TEC DCX DH. HI-TEC DCX Freeride. HI-TEC SLK DH 2000.
HI-TEC SLK Dual 2000. $\quad \underline{\text { HI-TEC SLK Freeride }}$ HI-TEC SLK Lite.

Jamis Dakar 97. Jamis Dakar 99. KHS DH.
$\underline{\text { Kona Mokomoko 99. } \quad \underline{\text { KONA STAB PRIMO }} \text { Lenz Revelation. }}$

Mongoose NX 8. $\underline{\text { RM 7 7 9 }}$
RM Switch 2001. Sintesi Python. Trek Fuel.
Specialized Big Hit Comp Specialized Big Hit Pro Trek VRX400.
${ }^{\circ} 03$.
Trek VRX400 LT. Trek VRX 185 B.

## Path Analysis.

Chapter V - Flawed Theories and Bogus Marketing.

# Theory, text, illustrations, and editing by Ken Sasaki. <br> 4-bar path analysis by Peter Ejvinsson. <br> Spanish Version translated by Antonio Osuna. <br> "Linkage" suspension simulation by Gergely Kovacs. 

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## Read these sections if:

You wish to understand why the following theories regarding bicycle suspensions are complete nonsense.

Skip these sections if:
You already know that most marketing, including these theories, are B.S.
The "'Brake Induced Shock Lockout' (BISL)." section is not difficult.
The "'Pivot at the Chain Line" (PCL)." section is also not difficult, as one must simply consider the problem depicted in Figure 2.5) or accept the results in "An Intuitive Look at Forces and Torques.".

Some of the "Ellsworth's 'Instant Center Tracking' (ICT)." section gets involved, but the parts where we use the different suspensions from the "The Natural Mirror Bike." section demonstrate some of the problems very simply.

The "'i-Drive" - a Perpetual Motion Machine?!?!" section is not difficult at all.
The "'Chain line does not matter in a URT'." section is moderately difficult.

The "Bogus Marketing." section is flat out easy.
But beware; the "False Claims for Floating Brakes." section is the most difficult in the work. An extremely strong physics background will be needed to tackle the last false theory in this section.

As far as we can determine, Mountain Bike Action magazine (MBA) espouses "Brake Induced Shock Lockout" (BISL)., "Special Point" Theories., and "Internal Force" Theories.

It is our purpose to educate the public on these matters and MBA reaches a fairly wide audience. We thus feel it important that MBA come to correctly understand bicycle rear suspensions. To this end, we have made numerous attempts to contact Richard Cunningham and MBA, as well as having sent them this work. We have met with no success.

If any readers are acquainted with these parties, we urge them to contact Cunningham or MBA to bring these issues to their attention.

We will now apply PA and its underlying "Some Important Concepts." to examine some well accepted, but flawed, quantitative design theories and bogus marketing. One fundamental flaw in all of the pedaling theories is the misunderstanding or neglect of the wheel effects covered in the "Center of Mass" section. These theories treat the suspension as if the chain were directly connecting the frame members, rather then connecting through the rear cogs.

## "Brake Induced Shock Lockout" (BISL).

There is a very well established myth (well-propagated by the magazines) that mono-pivot shocks will lock under rear braking. For example, in Mountain Bike Action magazine, Richard Cunningham states, "Most, if not all, mono-shock rear suspensions lock up under braking." (Page 76, Mountain Bike Action, May 2001). This is known as "Brake Induced Shock Lockout" (BISL).

Given the suspension dynamics we have covered, a little thought should make it obvious that application of the brake will have no "locking" effect on the shock, whatsoever.

Numerous theories have been put forward trying to establish the existence of BISL.
BISL theory \#1:
Perhaps the most amusing was the idea that if one locked the brakes on a mono-pivot and bounced up and down on the bike, then one would find the suspension would also be locked. The first flaw here is that while static friction between the wheels and the ground keeps the
wheels at a constant distance if the brakes and wheels are locked (neglecting fork compression), the same is not true while a bike is in motion. Static friction is lost either between the wheels and the brake or the wheels and the ground. Either condition will unlock the suspension. But beyond this, our plots of 4-bar paths in the "Typical Horst Link Designs." section show us that if mono-pivots are locking, then so too will most 4-bars, none of which are said to lock.

BISL theory \#2:
Some (see Richard Cunningham's quotes in the "Internal Force' Theories." section) believe 4-bars to brake better over bumps because braking forces are "isolated" on the rear link by the two rear suspension pivots. (This is an example of an "Internal Force" Theory, which we will further discuss in that section.) However, "Nature Varies Smoothly" (NVS) and the "Coaxial Condition" establish that pivots do not "isolate" forces. This theory is usually taken in combination with theory \#3.

BISL theory \#3:
Another widely accepted explanation for BISL, related to the last theory, is that application of the rear brake will push the swingarm down, causing the shock to compress, and thus stiffen or even lock against its bottom-out bumper. 4-bars are thought not to do this, since the force of the brake is not directly on the swingarm. Rather then the suspension supporting the main triangle, the main triangle is actually thought to hold up the rear and upper links!

The main thing that this explanation misses is that the front triangle will pitch forward under braking. The ultimate effect is that there is no significant shock compression or extension under braking in any of today's typical mono-pivot designs. This author has done a variety of tests on a number of mono-pivots to demonstrate that the shocks neither lock nor compress under braking. Bikes used in the tests included the now infamous, but really not so bad, Trek Y-bike (a URT, but equivalent under braking to non-URT mono-pivots). This bike was supposed to have one of the worst problems with BISL. We were fortunate in that one of the Y-bike shock pivots had a little squeak in it that was very noticeable under even very small movements (the squeak was not associated with any significant friction). From this, we had both visual and aural confirmation that the Y-bike shock was undergoing no compression or extension under braking.

As a final blow to BISL, note that we can construct a mono-pivot and a 4-bar that are both essentially neutral under smooth-surface braking and with the mono-pivot having, if anything, a slightly advantageous suspension rate under braking. In fact, the two 4bars that we are about to consider fit the bill precisely.

In addition to the errors we have pointed out in the above BISL theories, there are a number of suspension bike reviews in prominent magazines that indicate BISL to be a sham.

The Jamis Dakar series of bikes and the Psycle Werks Wild Hare are examples of bikes with IC's just about through the main pivots at equilibrium. They will thus be neutral under smooth braking. The upper links on these bikes also hang from the top tube, so they will experience an increased rise in rate under braking, as the suspension is compressed by some obstacle. This effect should be small, however, since the links start out at or near
perpendicular. If there is such a thing as BISL in mono-pivots, then these bikes should suffer from BISL to an essentially equal (or perhaps slightly greater) degree.

But we have seen numerous Dakar and Wild Hare reviews in Mountain Bike Action, Bicycling, and other industry magazines, with no mention of BISL as a problem. We find it particularly interesting that Mountain Bike Action, that BISL stalwart, would find no problems with these designs under braking.

Although there are small physical differences in typical 4-bars and equivalently main pivoted mono-pivots, we have yet to see any remotely reasonable explanation as to why one should brake better then the other in general.

The biggest consideration is the relation of the rider's body mass to the wheels and what it will do under braking. This author believes that between most of the designs, the differences are just not enough to merit a general statement.

Some people find 4-bars to brake better, but others do not, though we have seen no doubleblind tests. In the end, the small difference between some bikes may be significant enough for some people to feel a difference. But in general, we suspect that this is again just a case of the very well established psychosomatic phenomenon. This would not be the first time that people have been told that something is so and many have experienced what they have been told (this is why placebos cure illness). Or perhaps it is again a little of both.

We also have no doubt that the BISL myth has been propagated by some in the interest of selling more expensive 4 -bar designs. We see no $\$ 2,000$ mono-pivot frames.

In the near future, we hope to do a double-blind experiment to see once and for all if there is a difference. We will publish any results in subsequent editions of this work.

In any case, our advice here, as always, is for one to make decisions through testing the bikes, if possible.

## False Claims for Floating Brakes.

Beware; this section is the most difficult in the work. An extremely strong physics background will be needed to tackle the last false theory in this section.

We have seen that Floating brake systems give a bike the breaking character of its linkage geometry. There is nothing special in this beyond what is noted in the "Braking." section of Chapter III.

A number of theories out there claim advantages for floating brakes. But by this time, it should be trivial for the reader to prove most of these false.

For example, the idea that a floating brake will "isolate" braking from the suspension is as popular as the equivalent claim for 4-bar linkage suspensions [see BISL theory \#2 of the ""Brake Induced Shock Lockout' (BISL)." section]. This idea (also an "Internal Force" Theory) is also false for floating brakes for
the same reason it is for 4-bar suspensions. As stated in other sections, "Nature Varies Smoothly" (NVS) and the "Coaxial Condition" establish that pivots do not "isolate" forces.

However, there is one very alluring theory for floating disc brakes, purporting striking increases in performance. We must examine this, as it has swayed even some very technically sophisticated people. Before we tackle it, though, we must cover a little background.

When two objects are in contact, there is a force called friction that opposes any sliding movement between the objects. Each object produces a frictional force that acts on the other.

Consider, for example, Figure 5.1 A). Here is depicted a block sliding across a surface. The block has velocity V with respect to the lab, while the surface is stationary with respect to the lab. Depicted also is the normal force $\mathrm{F}_{\mathrm{N}}$, acting from the surface to the block.

Figure 5.1).


The frictional force between any two objects may be approximated with the following expression:

1) 2) $\mathrm{F}_{\mathrm{F}}=\mathrm{c} * \mathrm{~F}_{\mathrm{N}}$,
where is c is the coefficient of friction. This approximation is often very good and in the case of disc brake pads on a disk rotor, probably exceptionally good.
!!! Note: The following is very important. It is the key to understanding why the theory below is not correct. $\mathrm{F}_{\mathrm{F}}$ does not depend on the relative velocity of the objects. It only depends on the normal force. !!!

There are two types of friction coefficients; one for static friction (when the objects are not moving with respect to one another) and one for kinetic friction (when the objects have relative motion). We will refer to the first as "c(s)" and the second as "c(k)". c(s) is generally much greater then $\mathrm{c}(\mathrm{k})$. The product of the appropriate coefficient of friction and the normal force between the two objects gives the magnitude of the frictional force acting on either object, which is directed opposite any propensity for movement of either object, relative to the other.

In Figure 5.1 A ), we have labeled the frictional force " $\mathrm{F}_{\mathrm{Fl}}$ " and we see that it is directed opposite the motion.

In a brake, the brake pads apply a normal force to the disc (rim), inducing a force of friction that may be described by equation 1).

Let us now examine the very alluring claim for floating disc brakes that we mentioned above:

Consider a bicycle rolling over rough ground. The normal force between the tire and ground is greater on the front side of a bump then it is on the rear of a bump. This means that a larger frictional, or braking, force may be applied between the tire and the ground on the front side of a bump, without losing static friction.

It is thought that brake rotation counter to the wheel up the front side of a bump and an opposite rotation forward down the back side of a bump will apply just such a variable braking force between the tire and the ground. Floating brakes rotate the rear brakes in just such a way (see Figure 3.12). But can the claim be true?

First, note that a bicycle rolling along the ground under braking may be modeled very well by a block sliding across a surface. Kinetic friction on the frame is just removed one step from the ground by the wheel. If we consider the wheel as part of the "ground" system, then we see that the "bike" system (which now does not include the wheels) looks very much like a sliding block, with (almost) all of the friction concentrated at the brake pads. (If the brake is not applied at the rim, then torque will come into play, but this is irrelevant to the questions at hand.)

So to understand the braking bike situation, let's look at the much less distracting, sliding block situation.

We have seen in Figure 5.1 A) that a force of friction $\mathrm{F}_{\mathrm{F} 1}$ acts to slow the block down. But the frictional force is not dependent on the velocity. Therefore, if the block were sliding across the surface with velocity 2V, as in Figure 5.1 B), the result would be frictional force $\mathrm{F}_{\mathrm{F} 2}=\mathrm{F}_{\mathrm{F} 1}$.

Addition of velocities is linear in classical mechanics. We may thus switch reference frames and it is immediate that the situation in Figure 5.1 C), were the block slides with velocity V and the surface is now in motion with velocity -V , will produce frictional force $\mathrm{F}_{\mathrm{F} 3}=\mathrm{F}_{\mathrm{F} 1}=\mathrm{F}_{\mathrm{F} 2}$. It is also immediate that the acceleration of the block in all cases will be $a_{1}=a_{2}=a_{3}=F_{F 1} / \mathrm{m}$, where $m$ equals the mass of the block.

It is acceleration that we are interested in, after all, so we see that the rotation of the brakes back by floating brake systems is all for naught. This rotation back of the brake is analogous to the situation in Figure 5.1 C ), which produces the same negative acceleration as the situation in Figure 5.1 A ), which is analogous to a stationary brake.

Now floating brake proponents have countered with the following, involving the "energy" and "power" in the deceleration process.
2) 2) Energy $=$ Work $=($ Force $)($ Distance $)$.

Also,
3) 3) Power $=($ Energy $) /($ Time $)=($ Force $)($ Velocity $)$.

All of the above quantities are average values over time.
It is noted that rotating the brakes backward increases the arc length of brake rotor that goes through the brake pads. This increases the energy value over a given time span and thus the power.

This increase in arc length is thought to be similar to increasing the radius at which the brake pads operate, which also increases the arc length.

Floating brakes are thus said to be more "powerful" then stationary brakes.
It is true that energy and power are increased when considering the relation between the brake pads and brake rotor. But this is irrelevant.

To see this, consider again our block sliding on a surface in Figure 5.1). The block decelerates the same in all cases. But the length of ground sliding under the block per time is greater in B ) and C ) then it is in A ). This means that both the energy and power, as measured in the surface reference frame will be greater for $B$ ) and $C$ ) then it will be for $A$ ). But ultimately, it is the deceleration that we are concerned about. If the deceleration is not increased, then there is no advantage.

The above is a proof that Energy dissipated by a brake (the brake is the ground in this case) is irrelevant to the deceleration produced. But what is the problem fundamentally?

The problem, ultimately, is that energy is relative and not linear with velocity, and that those espousing floating brakes are only looking at one side of the process when reference frames are switched. The work done is increased, but so too is the relative energy. In the end, the two cancel out.

Put another way: The assertion is that there is something to be gained by the brake doing more work, while the bike/rider itself has no more kinetic energy with respect to the ground. But remember that the brake is moving with respect to the bike. This shift in reference frame will make the bike look like it has more kinetic energy, so nothing is gained.

An analogy may be drawn to our block sliding on the surface. If the block had a movable bottom that could be shifted back and forth relative to the main mass of the block, then the ground would "think" that the block had more kinetic energy while the bottom is shifted forward, in the direction of motion. But the deceleration would again be the same. (In fact, there would have to be "internal" energy released to shift the bottom forward, which is where the extra energy resides.)

As for increasing the radius at which the brake pads operate, that produces an increase in the lever arm and thus the torque. The increased braking force at the ground, for a given force to the brake lever, is due to this increase in torque and has nothing to do with the arc length of brake rotor traveling through the pads in a given amount of time.

Finally, one might think that the energy from compression of the suspension might be put into the brake and thus the bike would indeed have less energy compared to the work done by the brake, then in a fixed brake system. But this energy to compress the suspension ultimately comes from the kinetic energy of the bike. It is just transferred to the brake in a more roundabout way.

From an intuitive standpoint, consider whether or not your brakes stop you faster when you are traveling at a higher speed. If they did, brake modulation would be a very difficult thing.

It is really easy to do an experiment that will tell if the "Energy/Power" idea has any validity.

Construct a mechanism that will apply a constant amount of force too your rear brake lever; an elastic band should do fine. Then find out if it takes less time to slow from 30 mph to 25 mph then it does to slow from 10 mph to 5 mph .

In the first case, your brakes are doing far more work, but the deceleration will be the same.

## "Special Point" Theories.

The following two theories assert special points through which the chain line (or extended chain line) must pass, in order to make a suspension non-reactive to pedaling.

Traditionally this has been taken as true, regardless of mass distribution or any other considerations. However, as understanding of bicycle rear suspensions has evolved in the bike industry, some now take this to be true, given various conditions, for example conditions on mass distribution and pivot location.

We will consider only the more restrictive theories here, since if special point theories are false, even when mass and so forth is taken into consideration, then they will certainly be false more generally.

We begin this section with a general proof that all special point theories are false, even those accounting for mass. We will employ Path Analysis in this demonstration.

However, it will be instructive to state the special point theories below, with some further comments. The second theory has a well-known name attached to it, so we will put Path Analysis to the test against the second theory to see which is really right.
A) A) "Pivot at the Chain Line" (PCL).

This theory states that locating a mono-pivot pivot along the chain force line will eliminate suspension activation under pedaling. Most mono-pivot producers, including Santa Cruz, Marin, and Ellsworth's Aeon division produce bikes based on this theory. Some take this to mean with acceleration (Ellsworth) and some without (Santa Cruz). A notable exception is the Titus Loco Moto.

We have already given a general proof that all special point theories are false. However, since PCL is such a common theory in the bicycle industry, we wish to spare no effort in exposing the problems. In addition, a number of people with whom the author has had regular contact are interested in certain specific issues. So in Appendix A) "PCL Problems; Some Further Calculations.", we do some further specific calculations to dispel any lingering doubts as to the erroneous nature of this theory.

The one last issue worthy of mention here is an erroneous justification for PCL known as the "Locked Wheel Scenario". It is reasoned that the large amount of friction between the rear wheel and the ground, under pedaling, is equivalent to a large amount of friction in the bearings, leading to a situation essentially equivalent to a locked wheel. This idea is false, since friction between the wheel and the ground does not directly involve the swingarm, as does friction in the bearings. To see this, consider Figure 5.2).

Figure 5.2)


Figure 5.2 A ) shows our pole and wheel turned sideways, with the wheel resting on a scale. The scale will read the force between the wheel and the ground. Horizontal tension in the chain will not alter the scale reading. More to the point, friction exerted on the wheel (red), will also have no impact on scale reading, since it is directed horizontally. The same is true for friction acting on the scale (orange). There could be locking friction or no friction, and the scale reading would be the same because there are no vertical components of force involved.

Figure 5.2 B ) again shows a pole and wheel, but this time friction is created in the wheel via a horizontal pole rubbing against the wheel. The horizontal pole is attached to a vertical pole, which in turn is attached to the swingarm pole. Friction acting on the horizontal pole (orange), pulls the pole forward, creating a torque on the vertical pole about the point where it meets the swingarm. This
ultimately creates a torque on the swingarm pole about the green swingarm/wall axle. The vertical components of force occur where the vertical pole meets the swingarm pole. In this way, friction applied to the wheel feeds directly back into the swingarm, just as it would in the case of friction in the bearings. Horizontal chain tension will obviously lift the wheel, altering the reading on the scale.

Thus is dispelled the "Locked Wheel Scenario".
B) B) Ellsworth's "Instant Center Tracking" (ICT).

In 2002, a series of patents was granted to Ellsworth, describing a recipe for constructing bicycle rear suspensions, called "Instant Center Tracking" or ICT [examples include U.S. Patent 6,378,885 and U.S. Patent 6,471,230].

Before the distribution of Path Analysis and in the interest of fairness, we requested from Ellsworth a complete explanation of their ICT theory. Upon receipt of what Ellsworth claimed were the essentials of the ICT theory, we conducted an analysis and submitted it to Ellsworth and their consulting engineer Mike Kojima, so that Ellsworth could have a chance to express their opinions, and perhaps modify their theories and marketing, before we released the information. Our original public characterization of the Ellsworth claims and experimental work was taken almost verbatim from Ellsworth literature and correspondence.

We have since been able to review some of the Ellsworth patents and have found that, although significant conversations and exchanges of documents took place, regarding ICT, Ellsworth was not remotely forthcoming in explaining the details of their theory (we suspect that they feared fully informed and proper scrutiny of their theory). We have reviewed the correspondence and recalled that we continually urged Ellsworth to provide more detail for key elements of the theory, particularly the matters of "squat" and "anti-squat" (see below), if such existed. These calls were largely unmet, with Ellsworth claiming that they had disclosed the essentials of the theory. But again, we have now determined that forthright disclosure was not remotely the case.

All explanations considered, as well as examination of the bikes Ellsworth has produced over the years, seem to indicate that ICT has evolved over the years. Yet there are core problems that remain in all versions. We here present the problems that are common to all versions we have encountered.

Ellsworth marketing claims, on the basis of ICT, that their dual suspension bikes have "Up to $100 \%$ pedal efficiency (in every gear, and throughout the entire suspension travel range)". The "Up to..." phrasing is very confusing, but
through inquiries to Ellsworth we understand that it means that the bikes are almost $100 \%$ efficient in all gears.

In advertisements for the "Dare" downhill bike, Ellsworth has gone further, claiming that, "The 2001 Dare, with our patented ICT technology (which offers $100 \%$ pedal energy-efficiency by isolating pedal input from suspension activity), will out-accelerate, out-pedal and out-climb any full-travel free-ride bike on the planet." [See Page 22, Mountain Bike Action, May 2001.] No mention is made of gearing at all.

Ellsworth also claims that chain tension is decoupled from other forces on the suspension, so that these forces won't feed back through the chain to disrupt the rider's pedaling

Ellsworth further claims that their suspension is unaffected by braking forces.
We first present and examine the Ellsworth claims for pedaling, after which, we present and examine the claim for braking.

Ellsworth provides the following primary recipe for a 4-bar suspension, which is supposed to achieve their claims for performance, under pedaling:

Ellsworth first determines what they consider to be an average, extended chain line, through which chain force from the pedals passes.

In order to eliminate the effects on the suspension of force applied through the chain, from the pedals, Ellsworth claims that the linkage IC (with respect to the main triangle) should align with, or "track", the chain line, through all positions in the suspension travel. In the case of the Ellsworth Truth, the deviation from this ideal is said to be within $.5 \%$, at all times (that is, the deviation is small).

Ellsworth then assumes a particular rider/bike center of mass.
Ellsworth supposes two effects of this mass from bicycle forward acceleration, ultimately due to the force at the tire/ground contact point. These are squat (a suspension compressing effect), and anti-squat or jack (a suspension extending effect). Squat is the inertial resistance of the rider/main triangle to forward movement. Anti-squat is supposed to counter this. Ellsworth asserts a desirable range of IC locations along the chain line that is supposed to balance squat with anti-squat. We could find no place in the patents where Ellsworth states how they determined the desirable IC range of locations (third parties have informed us that the numbers were generated experimentally). The desirable range is delineated by a percentage scale.

The acceleration effects of squat and anti-squat, as well as the percentage scale, are partially explained below in a quote from U.S. patent 6,471,230. Figure 5.3) shows "FIG. 6 " of U.S. patent $6,471,230$, which is the diagram for the following explanation.

Figure 5.3)


The Ellsworth explanation is as follows:
The torque interaction between the pedaling-induced wheel driving force and the ground can also cause rider-energy-wasting suspension compression due to a torque moment transferred to the shock absorbing means via the suspension upper rocker arms and lower yoke. To counteract this moment, the suspension has approximately 10-20 percent anti-shock-absorbing means compression (or "antisquat") built into the suspension geometry. As illustrated in FIG. 6, this percentage may be calculated by drawing an imaginary line through the center of the rear wheel tire contact patch and the "instant center". Another imaginary line is drawn through the bicycle and rider unit combination's center of gravity, perpendicularly to the ground plane. The point where this line intersects the imaginary line
from the rear wheel tire's contact patch to the instant center is called the "anti-squat calculation point". The height distance in units of measure of the "anti-squat" calculation point to the ground is divided by the height distance in units of measure from the ground to the bicycle and rider unit combination's center of gravity. This number gives the percentage of "squat resistance" built into the rear suspension's geometry, where 100 percent equals full cancellation and zero percent is no cancellation.

Ellsworth has further stated a belief that an IC moving far out in front of the bike increases efficiency in a wider range of gears.

While some rationale is given in the patents for not having an IC located too far forward or too far back, the lack of any quantitative explanation for how the desirable range of IC locations was determined prevents a full, mathematical examination of errors in the ICT theory. However, this will not prevent us from demonstrating that the theory is nonsense.

So, Ellsworth has identified two types of pedaling effects: the chain force, and the acceleration effects of squat and anti-squat. The idea is that, if squat and anti-squat are made to balance out, then a lack of contribution from the chain force will create a suspension that is both non-reactive to pedaling and free from pedal stroke disrupting feedback.
(Note: one might wonder if chain tension is somehow a part of the anti-squat effect, since the physical components of this effect are never explained. But, again, Ellsworth is clear that one of the main objectives of ICT is that chain tension be decoupled from other forces on the suspension and Ellsworth even takes some pride in claiming that their suspensions do not use chain tension to counter squat, as some other designs do.)

ICT is supposedly based on sound classical physics, that is, the classical laws of nature. In classical physics, all sound theories based on the laws of nature must hold in their limiting cases, since Nature Varies Smoothly. Physicists routinely look at these limiting cases to see if their theories hold up, since these cases are often more intuitively obvious then the general cases, making them very good tests of the theory. And fortunately for us, we have just such cases.

The parallel, pp-coaxial, and wp-coaxial 4-bars, which were introduced in the "The Natural Mirror Bike" section are all limiting cases for the possible configurations of all 4-bars. Ellsworth should consider the wp-coaxial 4-bar to fall under the ICT prescription, since they use the configuration in one of their technical diagrams.

PA and ICT are in direct conflict. So we will subject these two theories to three tests, using our three limiting cases, as well as the calculations done in the "PCL Problems - Some Further Calculations" section, to see which of the theories holds up. After each test, where feasible, we examine the fundamental problem with ICT causing it not to hold up (we will not be able to fully do this in the last case, due to a lack of quantitative explanation for ICT dynamics).

## Test number 1:

As noted above, ICT identifies two types of forces: the chain force, and the acceleration forces of squat and anti-squat. How Ellsworth treats the chain force is the heart and soul of ICT. So let us examine whether or not Ellsworth has a proper understanding of this critical issue.

If we eliminate the acceleration forces, this will allow us to focus attention on the chain force. To do this, one can imagine a bike sitting on ice; when the rider pedals, the bike will not accelerate, so squat and anti-squat will be eliminated. Putting a bike in a trainer that clamps the rear wheel axle will similarly eliminate the acceleration forces.

One can also imagine eliminating the effect on the rider/main triangle from the unsuspended fork lowers and the front wheel, which Ellsworth does not directly address anyway. This can easily be done by suspending the fork uppers from the ground.

With acceleration removed, consider the following:
PA correctly tells us that the parallel and pp-coaxial 4-bars will behave identically, if the path tangents are identical.

In the case of the "parallel" 4-bar, ICT tells us that we should have a chain line parallel to the swing arm, since the IC is moving around at infinity. ICT and the "parallel/mono" natural mirror thus give us a "parallel" chain theory for nonaccelerating mono-pivots.

In the case of the "pp-coaxial" 4-bar, ICT tells us that we should have a chain line through the coaxial pivots. As a result, ICT and the "pp-coaxial/mono" mirror give us a "Pivot At the Chain Line" theory for non-accelerating monopivots.

ICT theory thus gives conflicting prescriptions for the same physical situation.
Furthermore, in the "PCL Problems - Some Further Calculations" section, we directly calculate the proper pivot location, for a non-accelerating mono-pivot,
using full-blown classical dynamics. We find there that the chain should neither be parallel nor at the pivot. The same will be true for our two 4-bars.

ICT's fundamental prescription of a chain through the IC thus gives erroneous results, even when taken in isolation from other forces Ellsworth considers.

PA thus passes this test, while ICT shows its first two flaws: inconsistency and incorrectness, having a fundamentally flawed central assertion.

Since ICT is inconsistent and incorrect, in the absence of acceleration and forces coming through the fork, when these forces are back in play, ICT must still be both inconsistent and incorrect.

This first Ellsworth failure is in not understanding the significance of the rear wheel for bicycles, particularly the "Center of Mass" issues discussed in the "Some Important Concepts" section. This is most serious, since it leads to a false primary assertion of IC at the chain force line. The following explains the fundamental physics that Ellsworth fails to understand.

Recall again the pole and wheel experiment diagramed in Figure 2.5). We can construct an analogous experiment for a 4-bar linkage. Figure 5.4) shows a 4bar rear suspension attached to the ground and balanced at equilibrium, with IC pictured. For visual convenience only, we have mounted the axle at the mid point of the top link (rear link on a bike) in a symmetrical linkage. This means that the tangent to the wheel path, at the point pictured, is horizontal.

Figure 5.4)


We put to Tony Ellsworth the question of where the rope should be pulled in the above contraption so that it does not fall. Ellsworth replied that this was the first step in testing their ICT technology. A cardboard and thumbtack model was constructed and experiments were done. An experiment on a linkage porch swing was also cited as an example of the situation in Figure 5.4) (which, in their reply, they suggested I perform for myself). Ellsworth concluded that the proper place to pull the chain in the above contraption is through the IC, if we wish the contraption not to fall. By this time it should be obvious that the IC is not the direction to pull the chain. As with the pole and wheel, one should again pull the chain (almost) vertically.

What Ellsworth has done is neglect the wheel! This same error in the context of rear suspension is what causes inconsistency and incorrectness in ICT.
B) Mike Kojima also seems unaware of the considerations involved with the chain force being on the wheel. In response to our statement in the "'Pivot at the Chain Line' (PCL)." section that the pole and wheel from Figure 2.5) "should also have cast some serious doubt on this theory (again, think of the earth as a very large front triangle)." Kojima stated:

## C) "His pole theory should make this more obvious!"

This seems to indicate that Kojima believes a rope pulled through the hinge would not cause the pole and wheel to fall.

Given Ellsworth's incorrect answer to the question and that Ellsworth considers this experiment equivalent to the linkage porch swing experiment, it is clear that Ellsworth does not grasp the significance of the wheel. If the chain force in a bicycle were going directly into the rear link, instead of through the rear wheel, then the chain force line would be the place to put the IC, to at least make ICT theory consistent and correct, in the absence of acceleration and effects coming through the fork. But since bicycles have wheels, ICT theory is inconsistent and incorrect, even in this case.

## Test number two:

Consider a parallel 4-bar, with upper and lower links parallel to the ground. As noted in the "The Natural Mirror Bike" section, if we imagine moving the IC back, by moving the two forward linkage pivots together, until they are coaxial and at the same height as the rear axle, producing a pp-coaxial 4-bar, we will have an identical situation to the parallel 4-bar. The parallel 4-bar IC is located as far out in front of the bike as you can get it. The pp-coaxial 4-bar has an IC located quite far back. (If one wants to object that the parallel 4-bar has no IC, then just imagine moving the two forward linkage pivots together a distance of one angstrom. One will then have a bike with a remote, forwardly located IC, that does not differ significantly from the pp-coaxial 4-bar.)

PA correctly states that the two situations are identical.

ICT claims that an IC moving far out in front of the bike provides greater efficiency in a wider range of gears. Clearly this is not the case for the two 4bars under consideration.

In addition, IC location and movement have no direct correlation to the performance of a wp-coaxial 4-bar (beyond suspension rate), including efficiency over a range of gears.

Since the Ellsworth bikes are essentially mono-pivots under pedaling, we see that they are no more efficient then typical mono-pivots.

## PA thus passes this second test, while ICT shows a third flaw: arbitrary nature.

This second, and most disturbing, Ellsworth failure is the fundamental misconception of what an IC is and what it does.

As we have noted, an IC moves relative to the front triangle as the suspension moves. A pivot does not. As a result, the IC does not control frame motion in quite the same way that a pivot would.

But Ellsworth and Ellsworth's consulting engineer Mike Kojima view the IC as a pivot. This is clear from the following statements Kojima makes in critiquing the early PA (the PA statements are in black, while Kojima's statements are in red:

Within any small segment of any non-URT suspension's travel, that suspension will behave like a mono-pivot, with pivot located along the line perpendicular to the tangent of the wheel path relative to the main triangle. That is, pedaling a multi-link at any particular position in the travel, at equilibrium for example, will be like pedaling a particular mono-pivot.

This is not true at all and is the point where the author errs. By multi-link he has to mean a true multi-link with the pivot below the axle. A pivot above the axle makes a multi-link a single pivot bike. A true multilink is actually a single pivot also, the single pivot being the instant center. The beauty of an IC bike is that the pivot can be placed in a less compromising point due to that point not being controlled as much by the frame packaging, because it is a virtual pivot point in space.

Accounting for friction and suspension rate, the reactivity of all nonURT suspension types will increase by practically the same magnitude as the gearing varies from ideal. That is, no geometry will be significantly more efficient in a wider range of gears then any other (though if the ideal is in the middle of the gear range, such a design will have a better average performance).

This is wrong, the non-reactive point can be made to include a bigger range of gears when a very long virtual swingarm can be made by placing the instant center well forward of the bike.

The first Kojima statement is somewhat amusing, since he first states that PA is wrong in saying that any non-URT (4-bars in particular) will behave like a mono-pivot through any small segment of the rear axle path, but then goes on to claim that "A true multilink is actually a single pivot..."

But more to the point, Kojima clearly believes that an IC far out in front of the bike acts like a pivot, producing "a very long virtual swingarm". The parallel 4bar definitively demonstrates that this is false.

## Test number three:

PA says that, at any point along the rear axle path about the main triangle, the tangent to the path is what determines the initial response of a suspension to pedaling. This means that there are an infinite number of IC locations, along the line through the rear axle and perpendicular to the path tangent, that will produce the same initial results.

ICT is in direct conflict with PA, claiming that each IC location, along the line perpendicular to the path, gives a different result in balancing squat with antisquat.

Again, IC location and movement have no direct correlation to the performance of a wpcoaxial 4-bar.

PA's claim that, for a given path tangent, IC location does not matter in a 4-bar thus properly characterizes the wp-coaxial 4-bar.

ICT's claim that one IC location is preferable to others clearly improperly characterizes the wp-coaxial 4-bar.

PA thus passes this third test, while ICT shows its fourth flaw: again, arbitrary nature.
Now note that the Ellsworth Truth and Dare are as close to wp-coaxial 4-bars as is practical without the rear pivot interfering with the cogs. In these two bikes, then, the configurations of the upper links will be of little consequence, beyond suspension rate. Their rear wheel axle paths will be very close to those of mono-pivots, even more so then the bikes plotted in the "Typical Horst Link Designs" section. Path Analysis tells us that they will thus have performance under pedaling and bump feedback that is almost identical to mono-pivots, with main pivots in the same places as those of the respective Ellsworth bikes (minus suspension rate of course).

It is rather ironic that ICT is arbitrary in a situation oh so close to the bikes that Ellsworth produces.

Tony Ellsworth disputes this, saying, "You say the Truth ict has a path very close to a single pivot. TI doesn't. Don't think I haven't drawn each. And if it is 'similar' your assuming that the amount of difference is not feel able or insignificant. Again, you are dead wrong."

It is true that each person must decide what is significant or "similar", and what is not, but it is obvious that the variation between the axle path of the Truth and the most similar mono-pivot is many orders of magnitude less then the radius of curvature. The reader may draw his or her own conclusions.

This third Ellsworth failure is due to the same deficiencies exposed in test number 2, as well as, almost certainly, the use of a dynamic model for determining squat and anti-squat that is just wrong.

It is impossible to more directly analyze the problems with Ellsworth's dynamic model for determining IC location along the chain line, since they do not give a quantitative account in the patents.

We have been given third party accounts of the dynamics behind ICT that are said to come directly from Ellsworth. This dynamic model is extremely simple. We know that this third party has had extensive contact with Ellsworth and some of his explanations included what were said to be Ellsworth quotes, from e-mail correspondence. In addition, some of the problems in the ICT qualitative theory of squat verses anti-squat correspond well with problems in the third party's dynamics, suggesting that the third party accounts do represent Ellsworth's theory. However, although there is some indication that the third party accounts do represent Ellsworth's theory and the dynamic model in the accounts is extremely simple to analyze, we feel an analysis is not appropriate without some further corroboration.

If we come into possession of Ellsworth's dynamic model, with some additional confidence that we have a correct and complete account, we will further analyze the problems with it at that time.

Nevertheless, we can get a further idea that the Ellsworth dynamics are problematic by conducting the following exercise:

Begin by considering the wp-coaxial 4-bars in Figures 5.5 A and B ), which are identical, aside from the upper, forward pivot location, giving a different IC location. As is always the case with wp-coaxial 4-bars, the two configurations would perform identically (suspension rate aside).

Figure 5.5)


Now, suppose on both of the above suspensions, we imagine: increasing the rear wheel radius; correspondingly decreasing the rear cog radius; and keeping everything else the same. This produces the configurations in Figures 5.6 A and B ). ICT does not ascribe any significance to the location of the rear axle, so supposedly nothing has changed in either case.

Figure 5.6)


If ICT were correct that location of the rear axle does not matter, then the suspensions in Figures 5.6 A and B) must also be identical. But, as is shown, only one of them could have a chain through the IC, for a given front chain ring and BB location. ICT thus claims that they are different. So we see yet another problem in that ICT is again in conflict with itself.

## But there is no limit to the possible configurations we can create with this exercise. We have thus shown that ICT is incorrect in all non-limiting cases.

All of this indicates that, whatever dynamics Ellsworth is using to determine IC location along the chain line, it is really screwy.

Everything that we have done here requires Nature Varies Smoothly, in order to examine the limiting cases and, in the last demonstration, to use them as a reference for examining the non-limiting cases.

Numerous ICT (and PCL) adherents have formed psychological blocks to accepting the fact that Nature Varies Smoothly, in clinging to their theories. This appears to effectively include Ellsworth and Kojima, who utterly rejected the limiting case analyses at the time of the discussions and, we are told, continue to not accept the validity of examining limiting cases.

In our original ICT analysis, we explained:
\{An aside: One may ask, "Do our parallel and pp-coaxial 4-bars fall under the ICT prescription?" Put mathematically, our two 4-bars are limit points in the space of ICT 4-bar bikes. ICT 4-bars come infinitesimally close to our two 4bars, therefore, our two 4-bars impact on ICT theory to the same extent as any ICT 4-bar, regardless of whether or not one wants to define them as such. Looking at the parallel 4-bar in a practical way; the upper and lower links cannot be exactly parallel in any real world bike, so any such bike will in fact have an IC very far away (about half in the forward direction). Trying to establish a parallel chain line will put the chain line through this IC as accurately as any chain line can be through any IC on any bike. For the pp-coaxial" 4-bar, the coaxial pivot is indeed a true IC.\}

To this, Mike Kojima replied:

It is rather shocking that someone, said to have an engineering degree, would not understand these very elementary mathematical and physical concepts.

In any case, ICT now seems to recognize mass, but clearly mass is still not handled properly, as fundamental problems remain.

The above demonstrations, any one of which is fatal, expose Ellsworth failures in understanding the physics of bicycle rear suspensions.

But there is yet another problem with ICT, involving claims for improved braking.

Ellsworth's recipe for a 4-bar suspension, to supposedly achieve their claims for performance under braking, is explained below in quotes from U.S. patent 6,471,230. Figure 5.7) shows "FIG. 2", etc. of U.S. patent 6,471,230, which is the diagram for the explanations to follow.

Figure 5.7)


Ellsworth explains the linkage configuration, leading to the claims of increased braking performance, as follows:
...reducing brake torque reactivity of the suspension system by positioning a brake about the rear wheel of the bicycle so that braking forces created by the brake acting on the rear wheel are nearly perpendicular to a straight line passing through the rearward pivot points of the upper and lower rocker arms, thereby reducing brake torque reactivity of the suspension system.

Ellsworth adds:
Motion ratio is improved by selecting a rocker arm length close to that of the lower swingarm. To provide desirable Brake Torque Isolation performance, the rear of the link must permit brake loads to
be imposed at near 90 degrees (nearly perpendicular) as described elsewhere herein;...

In summation, Ellsworth claims that a 90 deg. angle between the rear and horizontal links, as is nearly the case on the Dare downhill bike, will "isolate" the suspension from brake force.

Ellsworth explains the physical motivations for the 90 deg. linkage configuration in the following two quotes:


#### Abstract

All, or almost all, caliper brakes mount and function identically in the bicycle industry--they all must squeeze the rim in the same place, which creates a torque at that point which is then transferred into the frame at the connection points of the member on which they are mounted (for example, the shockstay 9) to the lower and/or upper swingarms (members 6, 7, and 8 in the preferred embodiment illustrated herein). As discussed elsewhere herein, if that force is aligned at a 90 degree angle, then there is no torque, and that force has no impact on the compression or extension of the suspension, etc. Any deviation from 90 degrees creates a torque moment that will pull or push the swingarms up or down-resulting in compression or extension of the suspension.


Ellsworth adds:
To prevent unwanted suspension movement, bind or preload under the forces of braking, the rear brake device (consisting of either a disc brake caliper or conventional bicycle rim brake) mounting point is attached to the rear wheel attachment upright. The angle of the rear wheel attachment upright to the upper rocker arms statically approaches 90 degrees in angle in a loaded condition, causing the torque moment induced by brake forces to be transferred into the forward frame assembly laterally with minimal horizontal torque component. This transfer of the brake forces thus will not have an extending effect or compressing effect on the shock absorber, leaving the suspension free to move horizontally when activated by wheel bump forces while the rear brakes are in operation. Positioning the rear suspension's instant center relatively close to the ground plane also helps the rear suspension's bump compliance under braking.

The second quote is very confusing, we believe due to some mistakes in word usage. Patent wording is usually very thoroughly scrutinized, however, we believe that the reference to "horizontal" suspension motion was intended to reference vertical motion. In addition, the reference to "torque component" is probably non-technical, since any torque, about any suspension pivots, will always have a vector at 90 deg. to the plane of the bike. This last may be simply a matter of a non-technical writer.

If we accept the corrections to word usage in the second quote, Ellsworth seems to have an essentially correct understanding of the forces on the linkage directly from the brake. However, as both quotes make clear, they draw the wrong conclusions in believing these forces will not affect the suspension.

Ellsworth's error is in the belief that force directed exclusively down the axis of the upper link, would isolate the suspension from braking.

Recall Figure 3.14) of the "Braking." section. Imagine adjusting the links to create 90 deg. angles between the rear and horizontal links, as per ICT. It is true that the force F , from the brake, will be transferred directly down the axis of the upper link, however, this will obviously not prevent a reaction of the suspension. It is, after all, the rider/main triangle that is suspended. Force from the brake, directed along the upper link axis, will go into the main triangle, ultimately acting as an extending force on the suspension. This, in turn, will contribute to the rider/main triangle pitching forward, exacerbating the jack already caused by rider inertia.

As is covered in the text associated with Figure 3.14), it is not the angles between the rear and horizontal links that matters, but the IC location - too bad Ellsworth did not stick with IC location when it was the correct thing to do.

Imagine varying the angle between the rear and upper links, while holding the axes of the upper and lower links constant, producing a constant IC location under variation. The components of the force on the upper and lower links, from the rear link, are changing, but so too are the torque lever arms. In the end, this variation in angle will not change the brake's effect on the suspension. So the Ellsworth Dare would have the same braking character with a more conventional, much shorter, upper link.

Finally, in the ICT patents, Ellsworth makes a number of claims for the performance of "prior art" designs that we find very odd and worthy of note.

For example, referring to "High Single Pivot" bikes such as those, "Used by Foes, Mountain cycle, Bolder, Pro Flex, Cannondale, Marin, and others.", Ellsworth states:
"These designs are usually very brake-torque reactive, which causes the suspension to extend and lock out."

As noted in the "Braking" and ""Brake Induced Shock Lockout" (BISL)" sections, we have done extensive braking experimentation on the most common mono-pivots and found them to be very non-reactive - in fact, generally the least of all reactive - to braking forces. Numerous other riders doing similar experiments have echoed these results.

Referring to "Unified Rear Triangle" designs, "Used by Trek, Gary Fisher, Klein, Schwinn, Ibis, and others.", Ellsworth states:
"Depending on the pivot location, brake torque usually causes these designs to compress and pre-load or extend and lock up."

URT bikes are essentially the same as non-URT mono-pivot bikes, under braking. As noted above, we have extensively tested many mono-pivot bikes, including the infamous, but really not so bad, Trek Y-bike. The Y-bike was absolutely neutral under braking.

Ellsworth states of "Multilink, Low Main Pivot" designs, "Used by GT, Turner, Intense, KHS (the foregoing are all four-bar linkage designs) Ventana, Mongoose, and Diamond Back (the last three utilize a swing or bell crank linkage).":
"The wheel travels in a near vertical path, instead of an arc, thus increasing shock absorbing efficiency and reducing energy wasting wheel fore and aft oscillations."

This again seems to imply Ellsworth viewing the IC as a pivot. In any case, the rear axle path curvature of a 4-bar can be as described, but most have tighter curvature then most mono-pivots, as is demonstrated in chapter VI, "Wheel Path Analyses of Some Existing Models."

Ellsworth goes on to say:
"...currently most bikes using this design have been developed by trial and error with no clear understanding of all of the aspects of suspension function."

Given the information above, we find this statement highly amusing.

## "Internal Force" Theories.

The next two theories we consider are examples of what I call "Internal Force" theories. (Actually, we have already discussed two examples of this sort of theory in "BISL theory \#2" of the "'Brake Induced Shock Lockout' (BISL)." section and also the "False Claims for Floating Brakes." section.)

Suppose that we have a mechanism that has two parts connected by a pivot. An example is a URT bicycle. Internal force theories say that force interactions internal to one part, on one side of the pivot, do not influence the other part, across the pivot.

For example, it is sometimes thought that the chain force line between the crank and rear wheel in a URT does not have an effect on suspension reactivity to pedaling because the force is "internal" to the rear triangle (this is the second theory below). Another idea is that braking
forces are "isolated" on the rear link of a Horst link suspension and thus do not activate the suspension (see "BISL theory \#2" of the "'Brake Induced Shock Lockout' (BISL)." section and also the "False Claims for Floating Brakes." section). The same "isolation" on the rear link of a Horst link is said to be true for pedaling (we have not covered this, since we believe that it will be trivial for readers to repudiate, at this point). PA tells us that these ideas are false. In particular, "Nature Varies Smoothly" (NVS) and the "Coaxial Condition" establish that pivots do not "isolate" forces, as we have noted in numerous other places.

Mountain Bike Action (MBA), in particular, seems very taken with this false idea. The most convincing indication of this comes from the Richard Cunningham quotes below in the " $\mathrm{i}-$ Drive' - a Perpetual Motion Machine?!?!" section. Cunningham also makes some vague use of the terms "isolates" and "uncouples" in the context of chain stay pivot 4-bar suspensions [Page 70, Mountain Bike Action, May 2001]. For example, Cunningham says, "The Horst link isolates braking forces and chain tension in the seatstays and thus provides an active rear suspension." Regarding "Parallel link" suspensions (a type of 4-bar), he goes on to state, "The wheel is mounted to the vertical rear link, which uncouples it from the swingarm and delivers a truly active ride. You can pedal or brake over rocks and roots and the rear wheel will follow the terrain exactly as it does when you are coasting." (Here he assumes a disc brake mounted on the rear link.) It is not entirely clear what Cunningham means by "isolates" and "uncouples", but it is clear that these terms are used to describe an effect of pivot location. These quotes, in conjunction with the Cunningham quotes in the "i-Drive" - a Perpetual Motion Machine?!?!" section, seem to indicate that MBA espouses internal force theories.

We will apply PA to the i-Drive first, since the analysis is extremely simple. We will then give a rigorous force vector treatment of the URT chain line as a test for Path Analysis.

## A) A) "i-Drive" - A Perpetual Motion Machine?!?!

The i-Drive is produced by the GT bicycle company (Linkage data).
Information on the i-Drive, directly from high-level personnel at GT, has been extremely difficult to come by. Efforts to contact an authority from the GT product development department meet with no success and GT offered no useful information on its web site when we last checked.

However, we have spoken to both representatives of GT's tech support line and the director of race support recently. All explanations from these sources were consistent in asserting that the purpose of the mechanism is to keep the BB static with respect to the main triangle. Mountain Bike Action magazine (MBA) also has given this explanation in an article by Richard Cunningham [Page 83, Mountain Bike Action, June 2001], saying "The i-Drive eccentric allows the cranks to remain fixed in space as if they were bolted to the main frame as the rear suspension cycles." Indeed, this seems to be very close to the case by examination of the mechanism, so we feel reasonably confident about this much.

All explanations from GT personnel also asserted that chain tension was eliminated as a consideration, in the same way believed for a URT mono-pivot, because the BB is on the swing arm. MBA again echoes this [Page 83, Mountain Bike Action, June 2001]. Cunningham states, "The high pivot position adds big-hit compliance to the suspension. If the GT was a monoshock suspension, chain tension would lock out the suspension under power. Because the crank axle... is attached to the swing arm, this cannot occur." (Monoshock is an unfortunate name for a type of mono-pivot borrowed from motorcycle jargon).

Taking the i-Drive objective as keeping the BB static relative to the main triangle, we see that the i-Drive mechanism is all for nothing. To the extent that the i-Drive achieves this objective, its component paths are the same as those created by a simple non-URT mono-pivot with the main pivot at the same place as that of the i-Drive. As we did in the "The Natural Mirror Bike." section, we can create a mirror bike, this time with both an i-Drive side and a mono-pivot side. Neither of the mechanisms will interfere with the other. The movement of mass in the i-Drive is almost identical to a mono-pivot, the only (insignificant) differences being the movement of the eccentric on the swing arm and of the "dogbone".

It is very easy to see that, given the high main pivot position, the distance between the bottom bracket and the cranks will increase as the suspension goes through its travel. This means that pedaling will cause an extending force on the suspension in most (if not all) gearing, and there will be bump feedback to the pedals, just as in a mono-pivot. One wonders how Cunningham envisions these effects not to occur with a lengthening chain line between the cogs merely because the bottom bracket is on the swingarm.

Some time ago, I was talking to a physics professor who told me that a problem has developed in the United States Patent and Trademark Office (USPTO). Apparently the USPTO now has a problem in recruiting qualified people to examine and award patents. The professor told me that one of the clearest signs that this problem is very serious is that there is now a culture in the USPTO that espouses the viability of "Free Energy Devices", the most commonly known of which is the "Perpetual Motion Machine". Apparently numerous patents have been awarded for free energy devices.

Now although the i-Drive claims do not involve perpetual motion explicitly, if the i-Drive could indeed do as is claimed, then one could easily use it to construct a perpetual motion machine.

How to construct a perpetual motion machine:
The i-Drive is claimed to be "unaffected under power" [Richard Cunningham, page 83, Mountain Bike Action, June 2001]. This means that the suspension will not activate when the pedals are pushed, if the frame is part of a bicycle.

Now if we take an i-Drive frame and fix the main frame member (the one that defines the cockpit) to the ground, then the forces on the rear suspension when
the pedals are pushed will be different then when the frame is part of a bicycle. In particular, this is true at the rear axle dropout.

As we learned from the "Center of Mass" (CM). and "An Intuitive Look at Forces and Torques." sections, the tension in the chain and the force on the pedals from the rider's pedal stroke will be felt at the crank axle as parallel forces. These are the forces that act on the rear triangle, at the bottom bracket. This means that the $i$-Drive rear suspension will activate when the crank axle is pushed if the main frame member is fixed to the ground. But the bottom bracket will still not move with respect to the main frame member and thus the earth.

Energy is equal to force over a distance, or $E=F * d$. When we apply a force to the crank axle and the axle does not move, we are doing no work on the mechanism, since the distance is zero. But if we attach an appropriate mechanism to the i-Drive rear dropout, the i-Drive rear triangle will produce a force over a non-zero distance and thus do work on that mechanism, when we push on the crank axle.

Voila! A free energy device! From this we can create perpetual motion by feeding the energy from the i-Drive rear dropout, back through the attached mechanism, to produce more force at the crank axle.

All of this, as well as what we demonstrate in the "Ellsworth's "Instant Center Tracking" (ICT)." section, shows that a patent is no guarantee that a device will do what is claimed. As we have noted, a patent only requires a new idea, not that the idea actually work (to say nothing of overworked or inadequate patent examiners).

We have sent copies of this work with additional emphasis on the i-Drive to both GT and Richard Cunningham. We have also made numerous efforts to contact both parties in an effort to clear up their confusion. Neither party seems interested in a thorough examination of the problems in their theories.


#### Abstract

Now, it may be that GT got the main pivot point just right and that this is why some people seem to like it. But in any case, to the extent that the i-Drive achieves its objectives, the same results could have been obtained using a much lighter, simpler mono-pivot, with the main pivot in the same place as that on the i-Drive.


A footnote:
Ray Scruggs, an avid mountain biker, has done some measurements on a GT i-Drive. He says that the BB actually drops somewhat with respect to the main triangle, as the suspension compresses. This would smooth the pedal stroke for the forward or driving pedal, while increasing kickback for the non-driving pedal. All in all, this may tend to make the suspension feel as if the rider were pedaling a bike with a lower pivot and less kickback. But
this also would reduce the anti-squat from the chain. So the result is still no net advantage over conventional mono-pivot designs, for the dilemma of kickback verses anti-squat, with the complicated i-Drive mechanism. And the suspension is certainly not "unaffected" by pedaling, as GT and Cunningham have claimed.

However, very interestingly, if Mr. Scruggs' observations are correct, they may entail some significance for the i-Drive. With the i-Drive's very rearward tilting wheel path, there may be a significant increase in bump performance, but with possibly a less obtrusive kickback then what is normally associated with such a rear tilting wheel path.

So in the end, there may be some significance to the i-Drive mechanism, though it has nothing to do with what the manufacturer and magazines have claimed.

We have, as of yet, not duplicated Mr. Scruggs' measurements, though we know him to be a fairly careful man. Analysis of Linkage data indicates that the bottom bracket does drop just slightly, which would tend to support Mr. Scruggs' claim. However, the vertical movement is slight. There is actually much greater horizontal movement.

One certainly would not expect the mechanism to achieve its results perfectly, so we are not terribly surprised that there should be some movement of the bottom bracket. As is always the case, each person should decide for himself or herself whether or not the deviation is significant.

At this time, we think it best to keep the main expose on the i-Drive as it has been, based on the manufacturer's claims, while noting the above deviations.
B) B) "Chain Line Does Not Matter in a URT".

It is a common misconception to think of chain tension in a URT (and i-Drive) swing arm as "internal" to the swing arm and thus not relevant to interactions between the swing arm and other bodies (the effect of gearing on acceleration is either forgotten, or held to be a different issue). Path Analysis tells us that this is false. But let us examine the situation rigorously for a URT to see if Path Analysis is really right. Since virtually the entire bike industry believes that "internal" chain tension does not matter in a URT, proving the idea wrong should be a convincing test for Path Analysis.

We will look at two separate arguments here. In both cases, we assume that the wheel is of reasonable mass.

We will first make an argument based on the simpler example of the pole and the wheel, and two of the concepts integral to Path Analysis, "Nature Varies Smoothly" (NVS). and the "Coaxial Condition". This will give a simple, intuitive understanding that chain line due to gearing does matter in a URT. Unfortunately, we cannot think of a good intuitive explanation that even the same set of cogs will produce different results in different positions without resorting to "(torque) $=($ force $) \times($ lever arm $)$ ".

We will then perform a rigorous force vector analysis.

Start with the pole and wheel situation depicted in Figure 2.5) of the "Center of Mass" section, mounted on a stand. Now it does not matter what we use to pull the string. We could pull the string using a crank, with the axle of the crank mounted on the stand, almost coaxial with the pole hinge, but just below it.

Let us assume that the crank cog is much bigger then the wheel, so that the pole will fall right very surely. Let us also have the pedals at the three o'clock/nine o'clock position and apply force to the pedals by an impulse straight down (this is done only for simplicity of analysis). See Figure 5.8 A) for the diagram.

Figure 5.8)


Now suppose we move the crank axle up so that it is coaxial with the pole hinge, but still mounted to the bench [see Figure 5.8 B )]. By NVS, nothing has changed. The pole will still fall to the right very surely.

We may now mount the crank to the pole, with the crank axle being coaxial to the pole hinge. But by the Coaxial Condition, again, nothing has changed. The pole will still fall right very surely.

If we now slide the crank slightly up the pole, by NVS, the pole will still fall right [see Figure 5.8 C )].

A similar argument can be made for a small crank pulling to the left - that is, a different gearing.

If gearing (chain line) and other "internal forces" did not matter, then the pole in Figure 5.8 C ) would fall or not fall in the same manner regardless of the size of the crank cog. It should now be obvious that this makes little sense.

One can imagine that even a particular set of cogs will give different results depending on chain line, since the situation is likely to continue changing as we move the crank further up the pole. To see this explicitly, one must understand that $($ torque $)=($ force $) \times($ lever arm $)$. We get the total torque about the pivot by summing the contributions from the force at the wheel axle and the force at the crank axle times their respective lever arms. One can see by drawing a few pictures [see Figure 5.9)] that as we slide the crank up the pole, both the force components and the lever arms will change, and not in ways that will cancel.

Now that we have the basic idea, imagine a crank/main-pivot coaxial mono-pivot in any situation. By the "Coaxial Condition", it does not matter whether we physically have a URT or a non-URT. The gearing will matter to the same extent in both cases (and everyone is already convinced that it matters in the non-URT case). As we move the crank off of the pivot, onto the rear triangle, the gearing effects will start out equal to those of a non-URT and change steadily. So we see that the same considerations that apply to the pole and wheel apply to a bicycle as well, and chain line matters in a URT.

We will now pursue a force vector analysis. Be mindful of the Center of Mass concept as we go through this, it applies both to the crank and the wheel. The results will be exactly the same as above.

Recall Figure 5.8 C).
What we have here is just a simple URT rear triangle hinged to the ground. There are two basic cases that we can have for the chain line. It can be vertical, or it can be non-vertical. The question is: If we apply an impulse to the right side pedal straight down (again, we choose this merely for simplicity), will the direction of the chain line influence whether or not the pole will fall and in which direction?

We need to look at all of the forces on the pole for the two cases.
The impulse at the pedal induces forces at the crank axle and at the pole/ground hinge, and it puts tension in the chain. The tension in the chain induces forces at the wheel and crank edges, which in turn induce forces at the wheel and crank axles.

In both cases, the vertical impulse at the pedal is felt at the crank axle as a vertical force. This, in turn is balanced by other vertical forces from the ground and the vertical components of the forces induced in the chain and wheel. That is, all of the vertical components of forces cancel. This should be pretty
obvious, since we assume that the force of the pedal stroke does not dislodge the pole from the hinge or crush the pole. Now we must look at the non-vertical components.

We first analyze the forces for the case of a vertical chain line.
The chain tension is vertical and thus so are the induced forces at the crank and wheel axles. That is, all forces on the pole are vertical (there are no forces on the pole with horizontal components) and the pole will not fall.

But what about the case of a non-vertical chain line?
Here, the chain tension has a horizontal component and thus so will the induced forces at the crank and wheel axles. These are the only horizontal force components. The force at the crank axle will be opposite in direction to the force at the wheel axle and will generally not be of equal magnitude (due to the different inertias of the crank and wheel). These two forces also have different lever arms about the hinge at the bottom of the pole. As a result, a net torque will be induced around the hinge. The pole will fall in the direction of the force component at the wheel, assuming typical lever arms and a wheel of reasonable mass (without this assumption, we may actually get the opposite result). Again, all of the vertical forces cancel, as we expect.

We can see that even the same set of cogs will provide various results in various positions by moving a particular set around relative to the hinge and to each other. Figure 5.9) shows the forces on the pole for a particular set of cogs in a number of positions. Note the differences in the forces and lever arms in each of the cases, which make the torque about the hinge different.

Figure 5.9)


All this seems very strange at first. Here are some suggestions when thinking about this:

In our first example, the crank axle is almost coaxial to the pole/ground hinge, so the horizontal component of force at this point is largely cancelled by an (almost) equal and opposite force from the earth. That is, movement of the pole is restricted at this point. There is no such restriction of movement at the wheel axle, so that point will accelerate.

So we have established that chain line matters. We have used poles and wheels because they show very clearly how the forces apply. The results for particular chain lines will be different in a bicycle, but the same principles we have established here will apply, so the chain line will still matter in the same way.

Above all else, remember that when the pivot is close to the BB , the gearing (chain) situation in a URT will be very close to that of a non-URT.

## Bogus Marketing.

## Companies have to lie. Consumers expect us to say certain things and if we don't say them, then they will not buy our products.

The above is a very close paraphrase from a marketing executive working for one of the world's largest bicycle manufacturers (I cannot make it a quote, since there is a word or two that I am not $100 \%$ sure about, but the above is very close to the actual quote).

One must parse the words of advertisements very carefully because ads are often crafted to give a particular impression, while saying something completely different.

Perhaps the king of all slippery marketing phrases is the drug company mainstay, "Nothing has been proven to last longer - be stronger - perform better..." If you ask most people what this means, they will say that the referred-to product is proven to last longer etc. then everything else. The phrase actually means nothing of the sort. It says simply that no one has demonstrated the product to be worse then anything else - quite a different assertion. The product could in fact be the worst thing on the market; the phrase just states that no one has proven this.

We earlier looked at Ellsworth's marketing phrase, "Up to $100 \%$ pedal efficiency (in every gear, and throughout the entire suspension travel range)". I discussed this phrase with a professor of mechanical engineering at our local university. We agreed that the "Up to" at the beginning of the sentence makes the sentence so vague that it could mean almost anything.

However, unlike the drug company phrase, which is definitely crafted to deceive, we believe that the "Up to" phrase may just be the result of clumsy wording. Ironically, this is in part indicated because Ellsworth has gone much further in their advertisements, claiming " $100 \%$ pedal energy-efficiency" without any qualifications [see page 22, Mountain Bike Action, May 2001]. This last constitutes the most extreme interpretation of the "Up to" phrase, so Ellsworth obviously has no problem in making such an extreme claim directly.

To be fair, we must note that many companies make claims for no suspension bob and $100 \%$ efficiency. But without question, the most egregious example of bogus marketing we have ever seen regarding bicycle rear suspensions comes from Kona, in their ads for the King Kikapu and Mokomoko [see page 7, Mountain Bike Action, May 2001].

The ad claims, "SHOCK FUNCTION IS AFFECTED BY WEIGHT, FORCE AND GRAVITY - NOT BY PEDALLING_". One can tell that this was not written by anyone with any significant technical knowledge. An object's "WEIGHT" is a measure of the attractive "FORCE" between the earth and that object due to "GRAVITY". Your guess is as good as ours. (The language here is very reminiscent of those VW commercials where they claim that they get "a maxim amount of volume in a minimum amount of space" - !?!?)

Humorous wording aside, the phrase does claim no pedal activation of the shock, so we have a more serious issue to consider. Since the rear pivot is on the seat stay in these designs (more on this later) we know that the designs are essentially mono-pivots under pedaling, with the upper links acting as suspension tuning. We have proven directly that no mono-pivot is completely non-reactive to pedaling, so we know immediately that Kona the claim is certainly not true. For example, there are no qualifications for gearing.

The ad goes on to claim, "SHOCK IS MOUNTED IN LINE WITH SEAT TUBE ALLOWING SMOOTH SHOCK FUNCTION AND SUPERIOR SHOCK RESPONSIVENESS_". By "IN LINE", we suppose that they mean parallel to the seat tube. But there are an infinity of other directions that the shock could be mounted that would allow the same "SMOOTH" shock function (witness the Ventana Marble Peaks, and Rocky Mountains), so we ask, "What is the point?" We also ask, "'SUPERIOR' compared to what?" Not to any of the competing designs we are aware of anyway. Does Kona believe that a shock mounted out of plane to the frame is viable enough to merit a comparison?

Lastly, the ad claims that, "REAR STAY PIVOT MOUNTED ON SEATSTAY INSTEAD OF CHAINSTAY SO ALSO NOT AFFECTED BY PEDALING FORCES_". Here is the most abject bit of nonsense. What is not "AFFECTED"? Clearly the reader is to believe it is the suspension. Again, the "SEATSTAY" pivot essentially makes these bikes mono-pivots (upper link suspension tuning aside). This fact, in and of itself, is irrelevant to the degree with which the design is "AFFECTED" by pedaling forces.

Is this a deliberate attempt to mislead potential customers or just a case of extreme ignorance? We leave the answer to the readers. We simply conclude that when it comes to suspension ad mumbo-jumbo, Kona is King.

Another little trick we see now and then is the source-less quote. This is exemplified by Iron Horse, the company that brought us the G-spot [see the back inside cover, Mountain Bike Action, May 2001]. All in quotes, we have, "BEST ALL AROUND DESIGN", "TOP OF ITS CLASS", and "THIS BIKE IS A MUST HAVE". None of these have any attribution attached. The quotes make it seem as if there is some independent opinion being expressed, as is the general purpose of quotes in ads. Younger people especially, who are not experienced in looking for these things, are the most likely to be fooled. The company did not even bother to pay for a quote whore, as has become standard practice in the movie industry.

We conclude this section by reiterating our assertions regarding advertising in the "Main Conclusions.":

Our advice is to ignore all suspension theories and other claims put forth by frame manufacturers and industry magazines, and base your buying decisions exclusively on experimentation. That is, make your decisions by test riding the bikes, even if it is just a parking lot test (you can get a lot from a parking lot test). Ignore all marketing!

## Path Analysis.

## Appendices.

# Theory, text, illustrations, and editing by Ken Sasaki. <br> 4-bar path analysis by Peter Ejvinsson. <br> Spanish Version translated by Antonio Osuna. "Linkage" suspension simulation by Gergely Kovacs. 

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A) A) PCL Problems - Some Further Calculations.

Many with whom this author has regular contact have a particular interest in the following situation involving a bike not undergoing acceleration. In addition, as is pointed out in the "Ellsworth's 'Instant Center Tracking' (ICT)" section, this example has particular relevance in showing problems with ICT theory. For these reasons, in addition to PCL being so widely accepted, we will explore it a little further.

We proceed with a proof by counterexample to show that PCL is incorrect even for a bike undergoing no acceleration. We construct the counterexample as follows:

Recall that all of the forces on a coasting bike sum to zero. Bicycle acceleration acts on the frame members through the rear axle. So we eliminate the torques generated due to bike acceleration under pedaling by clamping the bike at the rear axle, in a friction wheel equipped trainer. We can dispense with the fork term by turning the bike sideways and imagining some alternative force to gravity keeping the rider attached to the seat. Figure A1) diagrams this:

Figure A1)


Next, for ease of visualization and calculation, we replace the frame members with a couple of poles at $90^{\circ}$ to each other. With the "main triangle" pole taken as having a reasonable, non-uniform mass distribution, this situation is not far from reality in many cases. As per our calculations in "An Intuitive Look at Forces and Torques.", we have left out the crank and will neglect the rotating parts of the rider's body (feet and lower legs). Figure A2) shows this situation:

Figure A2)


Looking at this picture, it should already be obvious that the pivot is not the right place for the chain, if we want the poles to stay at (or at least near) 90 deg. As the chain is tensioned, the two poles must rotate around the rear axle en unison to achieve no shock activation. If the chain is at the pivot, no torque will be on the main triangle pole to rotate it back.

But let's go ahead and calculate:
We neglect the centripetal acceleration created as the bodies start to rotate around the rear axle, since this is minuscule.

This leaves only the chain torque and the body interaction torques.
Since the poles are at $90^{\circ}$, the swingarm pole will not create an interaction torque on the main triangle pole. However, recalling that for every action there is an equal and opposite reaction, we see that the main triangle will create an interaction torque on the swing arm. Recalling the quantities from Figure 2.12) in "An Intuitive Look at Forces and Torques.", we express the torque equation for the swingarm as:

A1) $\mathrm{I}_{\mathrm{S}} * \alpha=\sum_{\mathrm{i}} \tau_{\mathrm{Si}}=\tau_{\mathrm{SC}}-\tau_{\mathrm{SI}}=\tau_{\mathrm{SC}}-\mathrm{m} * \mathrm{a} * \mathrm{~S}_{\mathrm{L}}$,
where $\tau_{\text {SI }}$ is the interaction torque from the main triangle, m is the mass of the main triangle, and " $a$ " is the main triangle linear acceleration.

Now $\mathrm{a}=\alpha * \mathrm{~S}_{\mathrm{L}}$, so we have:
A2) $\mathrm{I}_{\mathrm{S}} * \alpha=\tau_{\mathrm{SC}}-\mathrm{m} * \mathrm{~S}_{\mathrm{L}}{ }^{2} * \alpha$.

But we should recognize $m * S_{\mathrm{L}}{ }^{2}$ as a moment of inertia. That is, if we calculate an effective $\mathrm{I}_{\mathrm{s}}{ }^{\prime}$ as the inherent moment of inertia of the swingarm plus a moment induced by the mass of the main triangle, then we have:

A3) $\mathrm{I}_{\mathrm{S}} * \alpha+\mathrm{m} * \mathrm{~S}_{\mathrm{L}}{ }^{2} * \alpha=\left(\mathrm{I}_{\mathrm{S}}+\mathrm{m} * \mathrm{~S}_{\mathrm{L}}{ }^{2}\right) * \alpha=\tau_{\mathrm{SC}}$.
Or, with $\mathrm{I}_{\mathrm{S}}{ }^{\prime}=\left(\mathrm{I}_{\mathrm{S}}+\mathrm{m} * \mathrm{~S}_{\mathrm{L}}{ }^{2}\right)$,
A4) $\mathrm{I}_{\mathrm{S}}{ }^{\prime} * \alpha=\tau_{\mathrm{SC}}$.
Now we have an equation involving only the chain torque. One often finds that equations can be set up in a number of ways depending on how things are defined - in this case leading to a very interesting result. This also exemplifies the assertion made in regard to Equation 8) in "An Intuitive Look at Forces and Torques." that the torques will eliminate T in all expressions except that for the fork.

Following the previously done calculations for h in constructing Equation 8), we have:

A5) $\mathrm{h}=\frac{\mathrm{r}}{\left(1+\frac{\mathrm{I}_{\mathrm{S}^{\prime}}}{\mathrm{I}_{\mathrm{F}}}\right)}$.

We are left with only the first term from Equation 8) - that for the chain torque, as expected.

We can see that Equation A5) matches our intuition exactly. It tells us that the heavier the main triangle and rider are compared to the swing arm, the farther away we move from PCL. Even with only the chain force under consideration, the very common PCL theory could be true only if the mass of the rider/main triangle is zero! This again is in perfect accord with our intuitive study of Figure A2) above. The rider/main triangle is very large compared to the swing arm -about 40 times as large. Assuming a reasonable mass distribution for the rider/main triangle, one will find that $h$ equals about $80 \%$ of r .

After these calculations, it should be easy for anyone to verify that PCL is not correct, even for a rider pedaling a bike while floating in free space [again remember our pole and wheel experiment from Figure 2.5), with the earth as a very large front triangle]. In this case, an almost parallel chain line is appropriate.

The fundamental problem with PCL for non-accelerating bikes is the neglect of the wheel (the same neglect found in Ellsworth's ICT). If the chain were attached directly between the main triangle and swingarm, then the pivot would be the place to pull, neglecting acceleration. But then we would not have a bike. Because bicycles have wheels, PCL is incorrect.
B) B) Glossary.

At this time, the Glossary has been done to explain terms in the "Primary Concerns." chapter that may not be familiar to those new to mountain biking. We have not provided a detailed account of scientific terms in the later sections because of time constraints. We hope that those venturing into these sections will have adequate prior knowledge or know how to obtain such knowledge from more fundamental sources, or that the bold-written essential information will suffice to give a reasonable understanding. In the future, we hope to provide a more detailed account of scientific terminology.

Bob (suspension):
The tendency of a suspension to oscillate under pedaling from cyclic forces in the chain.

Bottom Bracket (BB):
The Bearing and axle mechanism supporting the cranks.
Coasting:
Riding ones bike while neither pedaling nor braking.
Cross-Country (XC):
A style of riding (bike) that involves all types of terrain - uphill, downhill, and technical riding.

Downhill (DH):
A style of riding (bike) that involves almost exclusively downhill terrain. Riders often use cars or ski lifts to get them to the top of the hill.

Equilibrium (suspension):
See "Sag".
Feedback (bump):
The tendency for the cranks to rotate backward due to an increasing chain length as the suspension compresses (due to a hitting bump for example).

Floating brakes:
Rear disc brake mechanisms in which the brake is mounted on its own linkage arms, which are not part of the load bearing rear suspension components. These mechanisms can give mono-pivots a braking character similar to those found on some 4-bars.

Freeride:
A style of riding (bike) that involves most of the same terrain as crosscountry, but with an emphasis on downhill and more aggressive maneuvers such as jumps or drop-offs.

Full Suspension Frame:
A frame that allows the rear wheel to move with respect to the rider. Usually this is accomplished through a system of levers supported by a spring or shock.

Geometry (frame or suspension):
The spatial configuration of frame members, pivots, and other components that make up a bicycle frame or suspension.

Horst Link:
Sometimes used to describe multi-link suspensions with lower rear pivots on the chain stays. Named after Horst Leitner of the now defunct Amp Bicycles, who patented a certain chain stay pivot location now owned by Specialized.

Kickback (bump):
See "Feedback".
Main Triangle:
In common non-URTs, defined by the seat, handlebars, and bottom bracket. In a URT, may also refer to the frame member defined by the handlebars and the seat.

Main Pivot:
The lowest and most forward of the pivots in any suspension mechanism. Responsible for handling the highest amount of side loading in the mechanism.

Mono-pivot:
A type of full suspension frame in which the suspension consists of a single arm or triangle, rotating about the main pivot.

Neutral Geometry:
Describes a suspension frame, configured such that the components do not move relative to one another during some action by the rider (usually pedaling or braking). A suspension will be neutral if a zero torque balance is maintained about its pivots in the presence of a particular action.

Qualitative:

Examination involving attributes, characteristics, properties, and other such "qualities, usually making little or no use of hard numbers.

Quantitative:
Examination involving hard numbers to describe relevant "quantities".

Rate:

See "Suspension Rate".
Sag (suspension equilibrium):
The position a suspension assumes when a rider sits on the bike but performs no action.

Soft-tail:
A limited-travel suspension design, typically with about 1.5 inches of travel, which has a shock, but no pivots. The frame material is usually titanium.

Squat:

The tendency of a suspension to compress during a pedal stroke, due to rider inertia. As the rear wheel rolls forward during the pedal stroke, the rider's mass will resist movement, causing the compression.

Suspension Rate (also "Spring Rate" or just "Rate"):
A function describing the force with which a spring will tend toward equilibrium at each point of compression or extension away from equilibrium.

Suspension Member:
The structural pieces of the suspension. In a mono-pivot, the swingarm is the only suspension member. In a 4-bar, the swingarm, rear link (seatstay link), and upper link comprise the suspension.

Swingarm:
The arm in any suspension mechanism that rotates around the main pivot.
Torque Balance:

The torque differential between two objects rotating around a common pivot. A torque balance of zero under some action means that the two objects form a neutral mechanism under that action.

Travel:
The vertical distance a suspension will move the rear wheel axle.
Type (suspension):
Various rear suspension classifications, defined for some propose, ex. 4bar, mono-pivot, Horst link...

## URT:

A type of mono-pivot full suspension frame in which the crank is mounted on the swing arm.

## Path Analysis.

## About the Authors.

Theory, text, illustrations, and editing by Ken Sasaki.

## 4-bar path analysis by Peter Ejvinsson.

## Spanish Version translated by Antonio Osuna.

"Linkage" suspension simulation by Gergely Kovacs.
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The following are short biographies and pictures for those who have contributed to the Path
Analysis project and wanted such information displayed.

## Ken Sasaki.

Ken Sasaki has been an avid biker for over thirty three years. As of this writing, he lives in Southern California, though that may change soon. His other hobbies include skiing (alpine and telemark), running, hiking, singing, and writing essays. He has a Bachelor of Science in mathematics, has worked in engineering, and is currently studying physics.

Other works by Ken Sasaki may be seen at the Ken Sasaki Web Site.


## Peter Ejvinsson.

Peter Ejvinsson is 30 years old and lives in Stockholm, Sweden. He has been riding mountainbikes for twelve years. He is also into skiing, rock climbing, kayaking, windsurfing, sailing and hiking. He has a Master of Arts and works as an industrial designer. He also has a degree in mechanical engineering.

Peter would like it noted that he is not sure that he agrees completely with all of Ken Sasaki's theories. There is a lot of it that he feels he does not understand. He asks the point be made that he has only produced the diagrams.

D) Gergely Kovacs

Gergely Kovacs is from Hungary. He is twenty six years old and works as a software developer and civil engineer. He has been a mountain biker for 10 years now, currently owning his second (or maybe the third by this time) fullsuspension bike. He started developing the Linkage software about two years ago, and now it's quite a nice analysis tool for techy mountain bikers.



[^0]:    Answer:
    You should pull vertically, to the extent that one may assume the mass of the earth to be effectively infinite. To be precise, one should pull almost vertically, but juuuuust slightly to the left of vertical for the pole not to fall, since the mass of the earth is not truly infinite (for those interested in why the line is not exactly vertical, consider conservation of angular momentum).

    Pull left of vertical and the pole will fall left, and analogously for the right (assuming the mass of the earth to be infinite). Pull through the pivot in particular and the pole will fall left. Figure 2.5) diagrams the situation.

